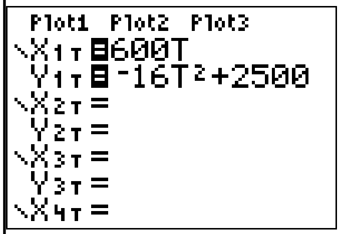
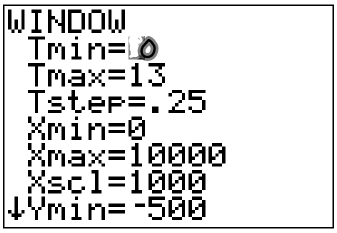
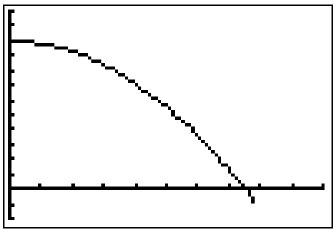
2.1 Rates of Change and Tangents to Curves

Consider an airplane at altitude 2500 feet flying 600 feet per second parallel to level ground on a windless day. It drops a package above point x and lands 7500 feet from point x 12.5 seconds later. Find the AVERAGE RATE of descent from 5 to 9 seconds and the INSTANTANEOUS RATE of descent at 9 seconds relative to time.

According to physics, $\begin{cases} x(t) = v_0 t + x_0 \\ y(t) = -\frac{1}{2} g t^2 + v_0 t + y_0 \end{cases}$ that is, $\begin{cases} x(t) = 600t \\ y(t) = -16t^2 + 2500 \end{cases}$





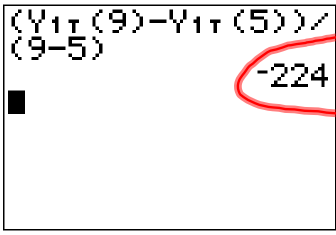


ymax = 3000
yscl = 250

Average rate of change is $\frac{f(b)-f(a)}{b-a} = \frac{\Delta y}{\Delta t}$

First, let's find the average rate of change in the package's height with respect to time from t = 5 to t = 9 seconds.

$$\frac{\Delta y}{\Delta t} = \frac{y(9) - y(5)}{9 - 5}$$



-224 ft/sec

Instantaneous : $\Delta t \to 0$

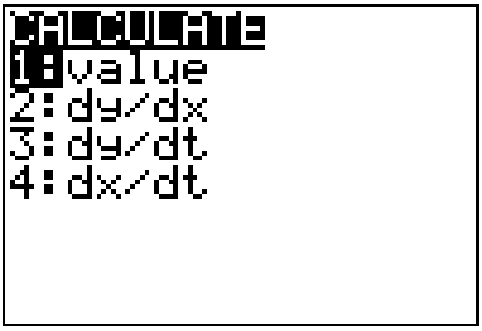
Approximate with $t = 8.999$ to $t = 9$

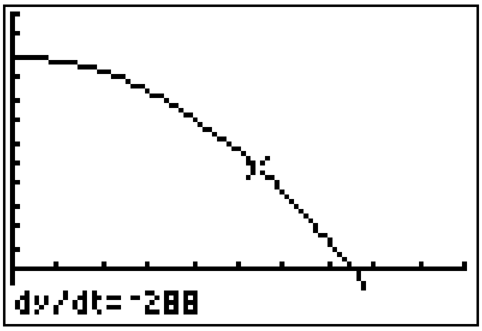
$$\frac{\Delta y}{\Delta t} = \frac{y(9) - y(8.999)}{9 - 8.999} = -287.984$$

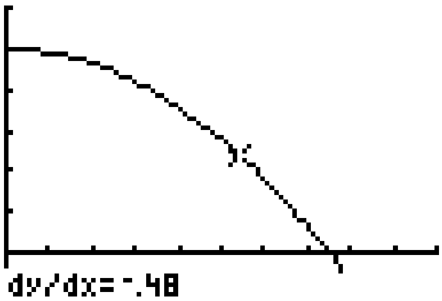
Derivative = instantaneous rate of change

is denoted $\frac{dy}{dt}$: $dy \Rightarrow \Delta y \rightarrow 0$

$dt \Rightarrow \Delta t \rightarrow 0$



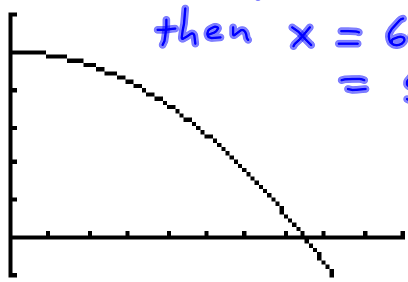




of course, the horizontal speed was given.

$$\begin{cases} x(t) = 600t \quad \cdots \quad t = \dfrac{x}{600} \\ y(t) = -16t^2 + 2500 \quad \cdots \quad y = -16\left(\dfrac{x}{600}\right)^2 + 2500 \end{cases}$$

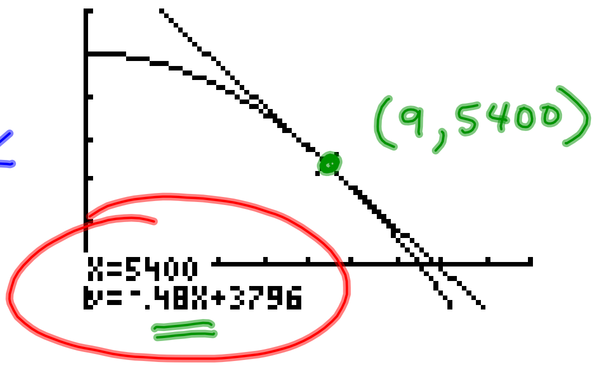
```
Plot1 Plot2 Plot3
\Y1=-16(X/600)²+
2500
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```



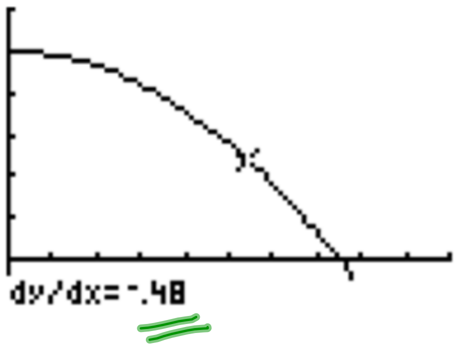
if $t = 9$
then $x = 600(9)$
$= 5400$

```
DRAW POINTS STO
1:ClrDraw
2:Line(
3:Horizontal
4:Vertical
5:Tangent(
6:DrawF
7↓Shade(
```

*



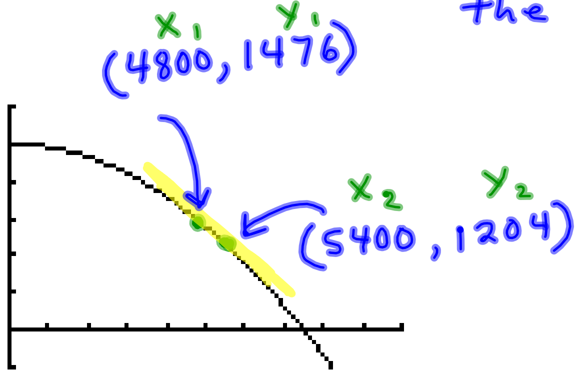
Result when we did this parametrically



Note: We can approximate tangent line with a secant line.

Try $\underset{t=8}{x_1 = 4800}$, $\underset{t=9}{x_2 = 5400}$

$\Delta x = 600$; the smaller Δx the better.



A secant line goes through Two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1204 - 1476}{5400 - 4800} \approx -0.45\bar{3}$$

$y = mx + b$; what is b ?

$1476 = -0.45\bar{3}(4800) + b \Rightarrow b = 3652$

$$y = \underbrace{-0.45\bar{3}x + 3652}_{\text{compare to } *}$$

a smaller Δx gets us closer to *

Check with regression.

Find the average rate of change of the function over the given intervals.

$f(x) = 5x^3 + 5$; a) $[4,6]$, b) $[-3,3]$

x's

```
Plot1 Plot2 Plot3
\Y1=5X^3+5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
(Y1(6)-Y1(4))/(6
-4)
```

$\frac{\Delta y}{\Delta x} =$ 380

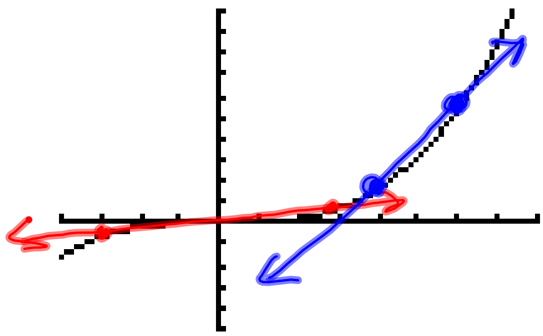
$$\frac{1085 - 325}{2}$$

```
(Y1(3)-Y1(-3))/(
3--3)
                45
```

$$\frac{140 - (-130)}{6}$$

```
WINDOW
 Xmin=-4
 Xmax=8
 Xscl=1
 Ymin=-1000
 Ymax=2000
 Yscl=200
↓Xres=1
```

The associated secant lines



Find the average rate of change of the function over the given interval.

$h(t) = \sin t;\ \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

```
NORMAL  SCI  ENG
FLOAT  0123456789
RADIAN  DEGREE
FUNC  PAR  POL  SEQ
CONNECTED  DOT
SEQUENTIAL  SIMUL
REAL  a+bi  re^θi
FULL  HORIZ  G-T
  ↓NEXT↓
```

```
Plot1 Plot2 Plot3
\Y1■sin(X)
\Y2=■
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
(Y1(π/3)-Y1(π/6)
)/(π/3-π/6)
         .6990570277
```

exact

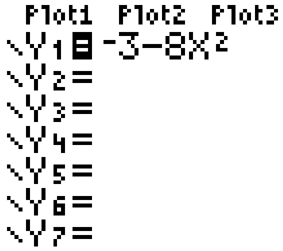
$$\frac{\sin\frac{\pi}{3} - \sin\frac{\pi}{6}}{\frac{\pi}{3} - \frac{\pi}{6}} = \frac{\frac{\sqrt{3}}{2} - \frac{1}{2}}{\frac{\pi}{6}}$$

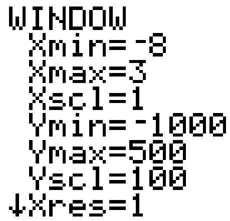
$$= \frac{\sqrt{3}-1}{2} \cdot \frac{6}{\pi}$$

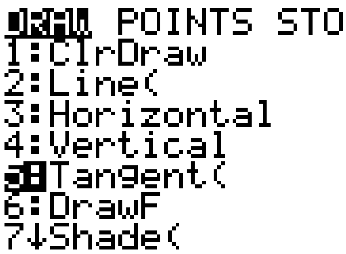
$$= \frac{3(\sqrt{3}-1)}{\pi}$$

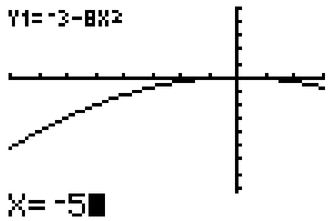
```
3(√(3)-1)/π
         .6990570277
```

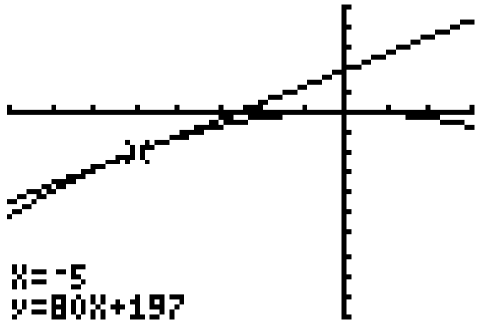
Find an equation for the line tangent to $y = -3 - 8x^2$ at $(-5, -203)$.











Slope of tangent line =

$\frac{dy}{dx} = 80$

y-int = 197

$y = 80x + 197$