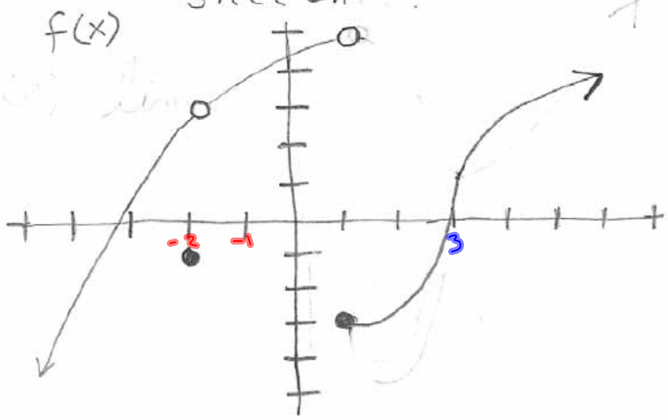
Limits - 2.2 and 2.4

Informal Definition of Limit: If x approaches c ($x \to c$, $x \neq c$), the limit of $f(x)$ is the y-coordinate L that $f(x)$ tends toward. We denote this with $\lim_{x \to c} f(x) = L$. In addition, the limit must be unique; the limit from the left must be equal to the limit from the right - i.e., $\lim_{x \to c} f(x) = L$ if $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$.

Ex. 1 Find the following from the sketch.



a) $\lim_{x \to -2} f(x) = 3$

b) $f(-2) = -1$

c) $\lim_{x \to 1^-} f(x) = 5$

d) $\lim_{x \to 1^+} f(x) = -3$

e) $\lim_{x \to 1} f(x)$ dne

f) $\lim_{x \to 3} f(x) = 0$

Ex. 2 Use your calculator to approximate $\lim_{x \to 5} g(x)$ where $g(x) = \dfrac{\sqrt{x^2-9}-4}{x-5}$

NORMAL FLOAT AUTO a+bi DEGREE CL

X	Y_1
5	ERROR
4.9	1.2573
4.999	1.2501
5.1	1.2432
5.001	1.2499

X=

$\lim_{x \to 5^-} f(x) = 1.25$

$\lim_{x \to 5^+} f(x) = 1.25$

$\lim_{x \to 5} f(x) = 1.25$

Ex. 3 Let $f(x) = \dfrac{4-x^2}{x+2}$

Find $\lim_{x \to -2} f(x)$

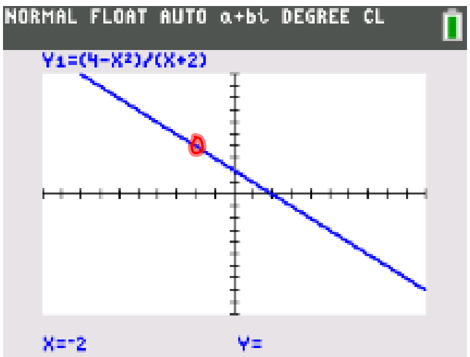
$\frac{0}{0}$ case

indeterminate form

$$f(x) = \frac{\cancel{(2+x)}(2-x)}{\cancel{x+2}} = 2-x$$

continuous

$$\lim_{x \to -2} f(x) = f(-2) = 2-(-2) = 4$$



NORMAL FLOAT AUTO a+bi DEGREE CL

X	Y_1
-2	ERROR
-2.001	4.001
-1.999	3.999

X=

Ex. 4 Let $f(x) = \begin{cases} 2x+5, & x < -1 \\ -x^2, & x > -1 \\ 5, & x = -1 \end{cases}$

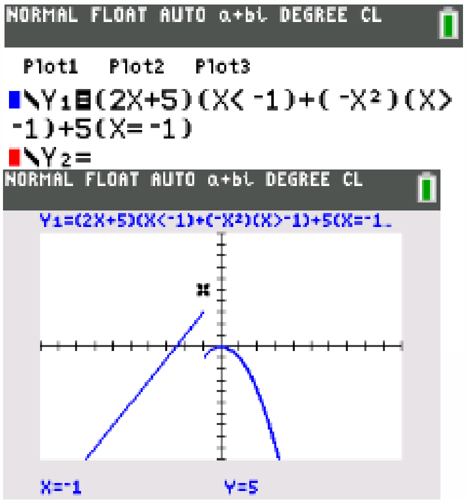
Find

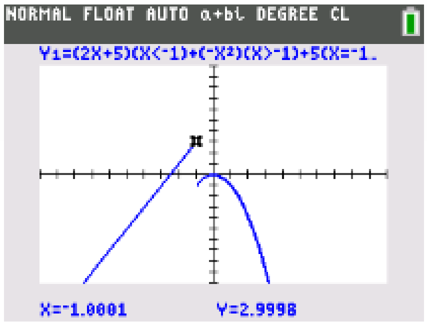
a) $f(-1) = 5$

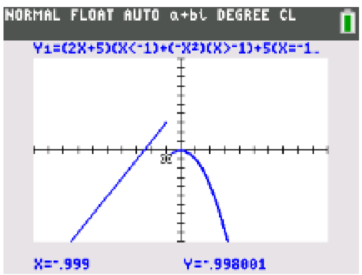
b) $\lim_{x \to -1^-} f(x) = 3$

c) $\lim_{x \to -1^+} f(x) = -1$

d) $\lim_{x \to -1} f(x)$ dne







Ex. 5 Let $F(x) = \begin{cases} x^2 - 1, & x < 2 \\ \sqrt{x+7}, & x \geq 2 \end{cases}$

Find

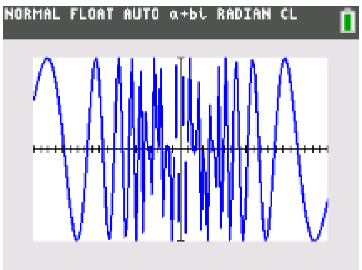
a) $F(2) = \sqrt{2+7} = \sqrt{9} = 3$

b) $\lim\limits_{x \to 2^-} F(x) = 2^2 - 1 = 3$

c) $\lim\limits_{x \to 2^+} F(x) = 3$

d) $\lim\limits_{x \to 2} F(x) = 3$

Ex. 6 Find $\lim\limits_{x \to 0} \sin\left(\frac{1}{x}\right)$



NORMAL FLOAT AUTO a+bi RADIAN CL

WINDOW
Xmin=⁻.1
Xmax=.1
Xscl=.005
Ymin=⁻1
Ymax=1
Yscl=1
Xres=1
ΔX=7.575757575757576E⁻4
TraceStep=.0015151515151515

NORMAL FLOAT AUTO a+bi RADIAN CL

X	Y_1
.01	⁻.5064
.001	.82688
⁻.005	.8733
⁻.009	.91494
⁻.015	.63902

X=

limit dne

oscillates

Theorem: If $f(x)$ is continuous on (a, b) and $c \in (a, b)$ then $\lim_{x \to c} f(x) = f(c)$.

Ex. 7 Find $\lim_{x \to 2} 3x^2 - 5x + 6$

$3x^2 - 5x + 6$ is a polynomial.
Polynomials are continuous on $(-\infty, \infty)$
$\therefore$ the limit is $3(2)^2 - 5(2) + 6 = 8$.

Definition: $f(x) = g(x)$ on (a, b) if the statement $f(c) = g(c)$ is true for $c \in (a, b) \sim K$ where K is a finite number of values in (a, b). $f(x) \equiv g(x)$ on (a, b) if $f(c) = g(c)$ for every c in (a, b).

Ex. 8 a) $\frac{x}{x} = 1$, but $\frac{x}{x} \not\equiv 1$ (why?)

b) $4x(x+1) \equiv 4x^2 + 4x$

c) $\frac{x+2}{x^2-4} = \frac{1}{x-2}$, but

$\frac{x+2}{x^2-4} \not\equiv \frac{1}{x-2}$ (why?)

Theorem: If $f(x) = g(x)$ then

$$\lim_{x\to c} f(x) = \lim_{x \to c} g(x).$$

Theorem usual applied when you have an indeterminate form such as $\frac{0}{0}$, $\frac{\infty}{\infty}$

Ex. 9 Find $\lim\limits_{x\to 3} \dfrac{2x^2 - x - 15}{7x - 21} = \frac{0}{0}$

$$\lim_{x\to 3} \frac{(2x+5)(x-3)}{7(x-3)} =$$

$$\lim_{x\to 3} \frac{2x+5}{7} = \frac{11}{7}$$

$\frac{0}{0}$

Ex. 10 $\lim\limits_{x\to 0} \left[\dfrac{\sqrt{x+1} - 1}{x} \cdot \dfrac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1}\right] =$

$$\lim_{x\to 0}\left[\frac{x+1-1}{x\left(\sqrt{x+1}+1\right)} = \frac{x}{x\left(\sqrt{x+1}+1\right)}\right] =$$

$$\lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{2}$$

NORMAL FLOAT AUTO a+bi RADIAN CL

X	Y_1
0	ERROR
.001	.49988
-.001	.50013

X=

Ex. 11 $\lim_{x \to 0} \frac{|x|}{x}$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\lim_{x \to 0^+} \left[\frac{|x|}{x} = \frac{x}{x} = 1 \right] = 1$$

$$\lim_{x \to 0^-} \left[\frac{|x|}{x} = \frac{-x}{x} = -1 \right] = -1$$

$\lim_{x \to 0} \frac{|x|}{x}$ does not exist