2.3 Definition of Limit.

$\lim_{x \to c} f(x) = L$ means that for every $\varepsilon > 0$, there exist $\delta > 0$ such that
(epsilon) (delta)

$0 < |x - c| < \delta$ implies $|f(x) - L| < \varepsilon$.

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note: $|x-c| < \delta \iff -\delta < x - c < \delta$
$\iff c - \delta < x < c + \delta$

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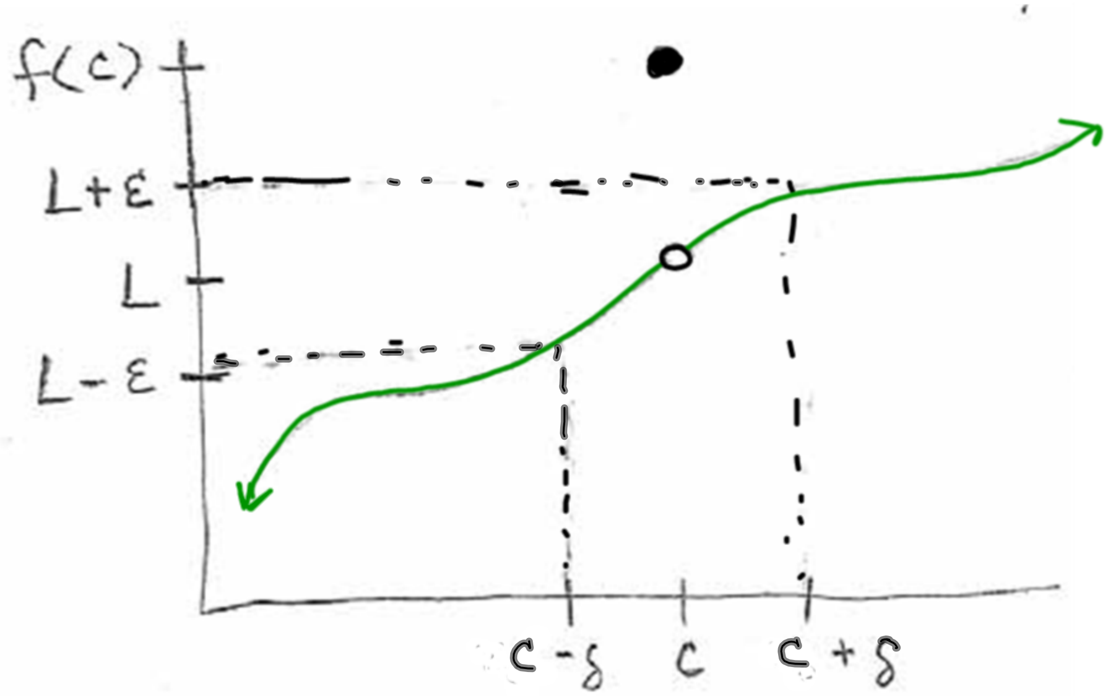
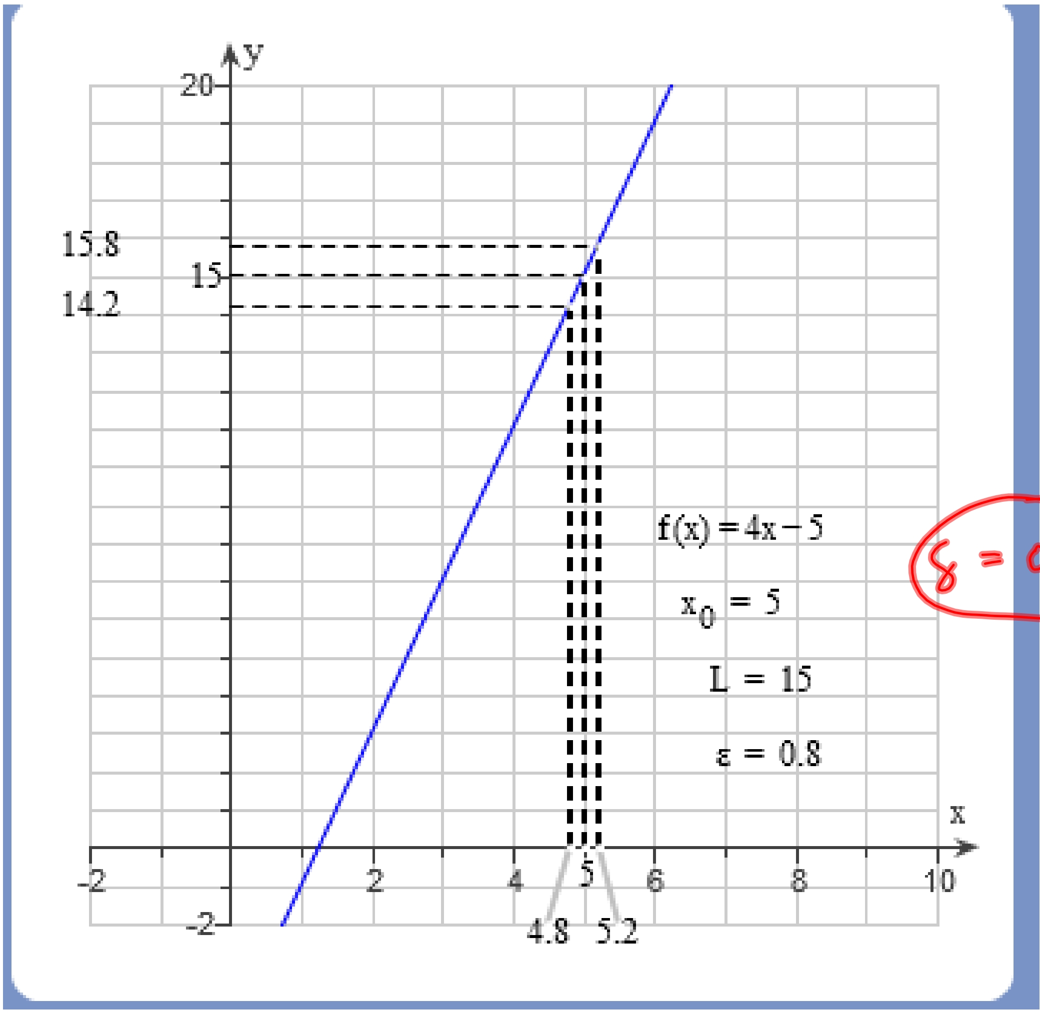
note: $|f(x) - L| < \varepsilon \iff -\varepsilon < f(x) - L < \varepsilon$
$\iff L - \varepsilon < f(x) < L + \varepsilon$



Use the graph below to find the maximum value of $\delta > 0$ such that for all x,
$0 < |x - x_0| < \delta \rightarrow |f(x) - L| < \varepsilon.$

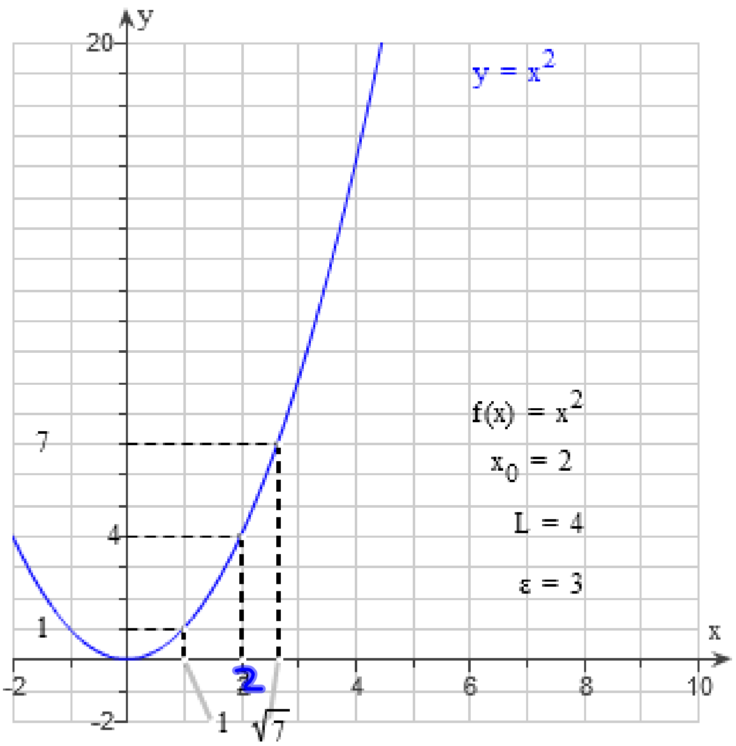
book uses x_0, I use c.

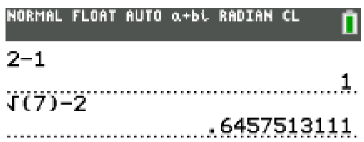


$\delta = 0.2$

Use the graph below to find the maximum value of $\delta > 0$ such that for all x,

$0 < |x - x_0| < \delta \rightarrow |f(x) - L| < \varepsilon.$





$\delta = \sqrt{7} - 2$

$f(x) = 6x + 3$, $L = 21$, $x_0 = 3$, $\varepsilon = 0.06$ $\lim_{x \to 3} f(x) = 21$

The largest open interval about x_0 on which the inequality $|f(x) - L| < \varepsilon$ holds is $(2.99, 3.01)$
(Use interval notation.)

The largest value of $\delta > 0$ such that $0 < |x - x_0| < \delta \rightarrow |f(x) - L| < \varepsilon$ is $\delta = 0.01$
(Simplify your answer.)

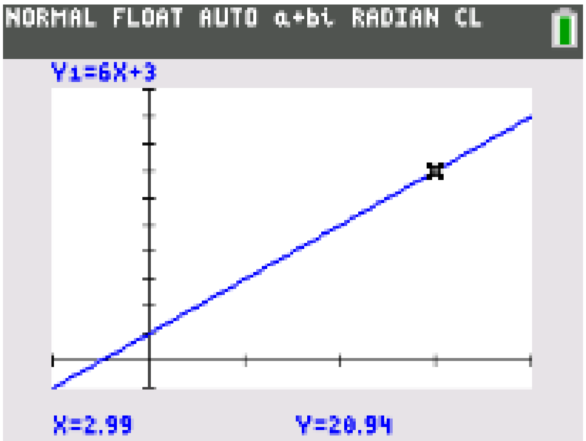
$$|f(x) - L| < \varepsilon$$

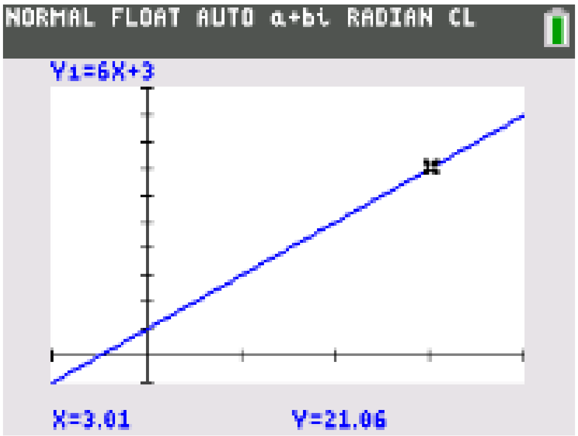
$$|6x+3-21| < 0.06 \iff |6x-18| < 0.06 \iff$$

$$6|x-3| < 0.06 \iff |x-3| < 0.01$$

$$\iff -0.01 < x-3 < 0.01$$

$$2.99 < x < 3.01$$





$f(x) = \sqrt{9x+76}$, $L = 11$, $x_0 = 5$, $\varepsilon = 0.05$

$\lim_{x \to 5} f(x) = 11$

The largest open interval about x_0 on which the inequality $|f(x) - L| < \varepsilon$ holds is (4.8781, 5.1225)
(Use interval notation. Round to four decimal places as needed.)

The largest value of $\delta > 0$ such that $0 < |x - x_0| < \delta \rightarrow |f(x) - L| < \varepsilon$ is .1219 .
(Round to four decimal places as needed.)

$$|f(x) - L| < \varepsilon$$

$$\left|\sqrt{9x+76} - 11\right| < 0.05$$

$$-0.05 < \sqrt{9x+76} - 11 < 0.05 \iff 10.95 < \sqrt{9x+76} < 11.05$$

$$119.9025 < 9x + 76 < 122.1025$$

$$43.9025 < 9x < 46.1025$$

$$4.8781 < x < 5.1225$$

$$\delta = \min\{.1219, .1225\} = .1219$$

Give an ε-δ proof of the limit fact.

$$\lim_{x\to 5}(3x-2)=13$$

$\varepsilon = .15$
$\delta = \frac{.15}{3} = .05$

For every $\varepsilon > 0$, let $\delta = \underline{\varepsilon/3}$.

Assume $\underline{|x-5| < \delta}$. This implies

$$|3x-2-13| = |3x-15| = 3|x-5| < 3\delta = 3\cdot\frac{\varepsilon}{3}$$
$$= \varepsilon$$

May 20-2:04 PM