Ex. 12 $\frac{0}{0}$ $\lim_{x \to -3} \dfrac{\frac{1}{x+5} - \frac{1}{2}}{x+3} =$

$$\lim_{x \to -3} \left[\frac{2-(x+5)}{2(x+5)} \div (x+3) \right] = \lim_{x \to -3} \left[\frac{-x-3}{2(x+5)} \cdot \frac{1}{x+3} \right] =$$

$$\lim_{x \to -3} \left[\frac{-\cancel{(x+3)}}{2(x+5)} \cdot \frac{1}{\cancel{x+3}} \right] = \lim_{x \to -3} \frac{-1}{2(x+5)} = -\frac{1}{4}$$

Ex. 13 $\frac{0}{0}$ $\lim_{x \to \pi} \left[\dfrac{\sin x}{1+\cos x} \cdot \dfrac{1-\cos x}{1-\cos x} \right] =$

$$\lim_{x \to \pi} \left[\frac{\sin x(1-\cos x)}{1-\cos^2 x} = \frac{\cancel{\sin x}(1-\cos x)}{\cancel{\sin x}\,\sin x} \right] =$$

$$\lim_{x \to \pi} \frac{1-\cos x}{\sin x} = \frac{1-(-1)}{0} = \frac{2}{0}$$

dne

NORMAL FLOAT AUTO a+bi RADIAN CL

Plot1 Plot2 Plot3

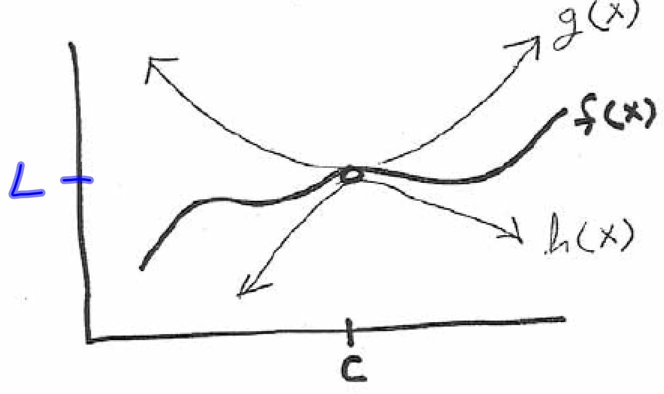
Y1=sin(X)/(1+cos(X))
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
Y9=

NORMAL FLOAT AUTO a+bi RADIAN CL

X	Y1
3.1416	ERROR
3.1	48.078
3.14	1255.8
3.1416	753861

X=

Squeeze Theorem (Sandwich Theorem)



If $g(x) \le f(x) \le h(x)$ for every x in an open interval containing c, except possibly c itself and

$$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L \text{ then}$$

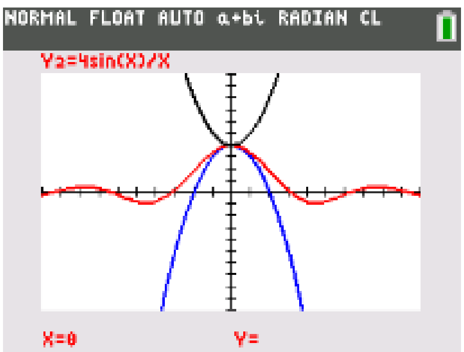
$$\lim_{x \to c} f(x) = L.$$

Ex. 14 Given $4 - x^2 \le \frac{4 \sin x}{x} \le x^2 + 4$

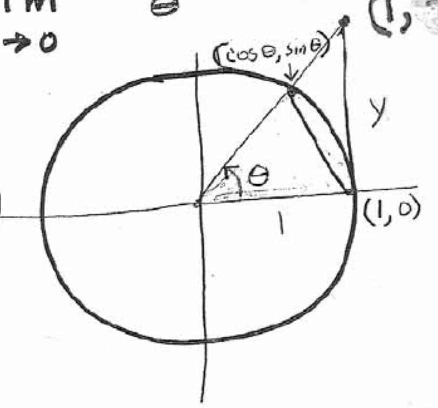
Find $\lim_{x \to 0} \frac{4 \sin x}{x} = 4$

$$\lim_{x \to 0} 4 - x^2 = 4$$

$$\lim_{x \to 0} x^2 + 4 = 4$$



Claim: $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$



$\tan\theta = \frac{opp}{adj} = \frac{y}{1}$

$0 < \theta < \pi/2$

proof that $\lim_{\theta \to 0^+} \frac{\sin\theta}{\theta} = 1$

Area of right triangle : $\frac{1}{2} bh$

$$\frac{1}{2}(1)\tan\theta = \frac{\tan\theta}{2}$$

Area of sector : fractional part of circle

$$\frac{\theta}{2\pi} \cdot (\pi r^2) = \frac{\theta}{2\pi}(\pi) = \frac{\theta}{2}$$

Area of triangle inside sector : $\frac{1}{2} bh$

$$\frac{1}{2}(1)\sin\theta = \frac{\sin\theta}{2}$$

$$\frac{\tan\theta}{2} > \frac{\theta}{2} > \frac{\sin\theta}{2} \Rightarrow$$

$$\tan\theta > \theta > \sin\theta \Rightarrow (\div \sin\theta)$$

$$\frac{1}{\cos\theta} > \frac{\theta}{\sin\theta} > 1 \Rightarrow$$

$$\cos\theta < \frac{\sin\theta}{\theta} < 1.$$

$$\lim_{\theta \to 0^+} \cos\theta = \lim_{\theta \to 0^+} 1 = 1$$

$$\therefore \lim_{\theta \to 0^+} \frac{\sin\theta}{\theta} = 1.$$

Similarly, we can show $\lim_{\theta \to 0^-} \frac{\sin\theta}{\theta} = 1.$

Claim: $\lim\limits_{\theta \to 0} \dfrac{1 - \cos\theta}{\theta} = 0$

$$\lim_{\theta \to 0} \left[\frac{1-\cos\theta}{\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta} = \frac{1-\cos^2\theta}{\theta(1+\cos\theta)} = \right.$$

$$\left. \frac{\sin^2\theta}{\theta(1+\cos\theta)} = \frac{\sin\theta}{\theta} \cdot \frac{\sin\theta}{1+\cos\theta} \right] =$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} \cdot \lim_{\theta \to 0} \frac{\sin\theta}{1+\cos\theta} =$$

$$1 \cdot \frac{0}{2} = 1 \cdot 0 = 0.$$

Ex. 14 $\lim_{x \to 0} \frac{\sin(5x)}{3x}$

let $u = 5x$
$u \to 0$
$x = \frac{u}{5}$

$$\lim_{u \to 0} \left[\frac{\sin u}{3 \cdot \frac{u}{5}} = \frac{\sin u}{\frac{3}{5} u} = \frac{5}{3} \cdot \frac{\sin u}{u}\right] =$$

$$\frac{5}{3} \lim_{u \to 0} \frac{\sin u}{u} = \frac{5}{3} \cdot 1 = \frac{5}{3}$$

Ex. 15 $\lim_{x \to 0} \frac{\tan(5x)}{x}$; $u = 5x$, $x = \frac{u}{5}$

$$\lim_{u \to 0} \left[\frac{\tan u}{u/5} = 5 \cdot \frac{\tan u}{u}\right] =$$

$$5 \lim_{u \to 0} \left[\frac{\tan u}{u} = \frac{\sin u}{\cos u} \cdot \frac{1}{u} = \frac{\sin u}{u} \cdot \frac{1}{\cos u}\right] =$$

$$5 \left[\lim_{u \to 0} \frac{\sin u}{u} \cdot \lim_{u \to 0} \frac{1}{\cos u}\right] = 5 \cdot 1 \cdot 1 = 5$$

Ex. 16 $$\lim_{\theta \to 0} \left[\frac{\sin \theta}{\sin 3\theta} \cdot \frac{\frac{1}{\theta}}{\frac{1}{\theta}} = \frac{\frac{\sin \theta}{\theta}}{\frac{\sin 3\theta}{\theta}}\right] =$$

$$\frac{\lim_{\theta \to 0} \frac{\sin \theta}{\theta}}{\lim_{\theta \to 0} \frac{\sin 3\theta}{\theta}} = \frac{1}{3}$$