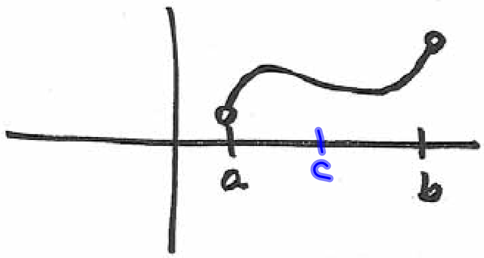
2.5 Continuity

In layman's terms, a function $f(x)$ is continuous on the open interval (a, b) if you can draw the function without picking up your writing utensil from a to b.



Formally, $f(x)$ is continuous on (a, b) if for every c in (a, b) <u>all</u> of the following are true

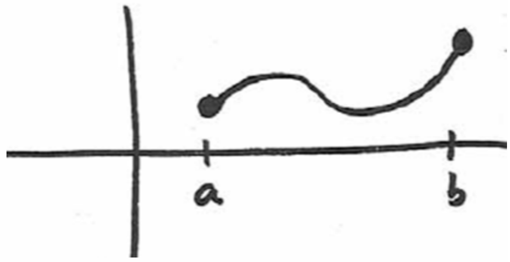
1) $f(c)$ is defined
2) $\lim_{x \to c} f(x)$ exists
3) $\lim_{x \to c} f(x) = f(c)$

If one of these is not true then c is called a point of discontinuity.

If in addition,

4) $\lim_{x \to a^+} f(x) = f(a)$

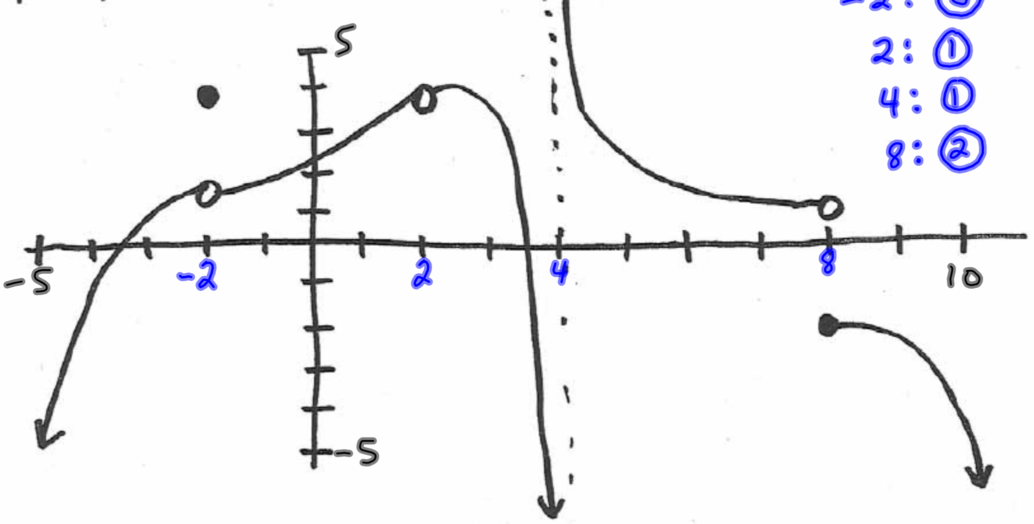
5) $\lim_{x \to b^-} f(x) = f(b)$



then $f(x)$ is continuous on $[a, b]$

Ex. 1

Find the point(s) of discontinuity. State the property or properties it violates.



-2: 3

2: 1

4: 1

8: 2

*Also, write the interval for which $f(x)$ is continuous.

$(-\infty, -2) \cup (-2, 2) \cup (2, 4) \cup (4, 8) \cup (8, \infty)$

Ex.2

At what points is the function $y = \frac{x+2}{x^2-3x+2}$ continuous?

$$= \frac{x+2}{(x-2)(x-1)}$$

disc at $x=1$, $x=2$

$$(-\infty, 1) \cup (1, 2) \cup (2, \infty)$$

Ex.3

At what points is the following function continuous?

$$f(x) = \begin{cases} \frac{x^2-4x-5}{x-5} & x \neq 5 \\ 6 & x = 5 \end{cases} = \begin{cases} \frac{(x-5)(x+1)}{x-5}, & x \neq 5 \\ 6, & x = 5 \end{cases}$$

$$= \begin{cases} x+1, & x \neq 5 \\ 6, & x = 5 \end{cases}$$

$$\lim_{x \to 5} f(x) = 6$$

$$f(5) = 6$$

no pts of disc.

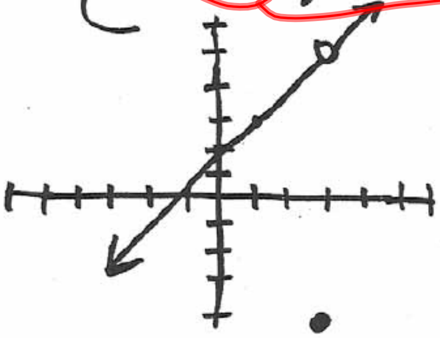
Continuous on $(-\infty, \infty)$

May 18-10:05 AM

If a function can be made continuous on (a, b) by redefining one point c then c is a point of removable discontinuity.

$$f(x) = \begin{cases} x+2, & x \neq 3 \\ -4, & x = 3 \end{cases}$$

if this were 5 this would be a continuous function on $(-\infty, \infty)$



$x = 3$ is a point of removable discontinuity.

Ex. 4

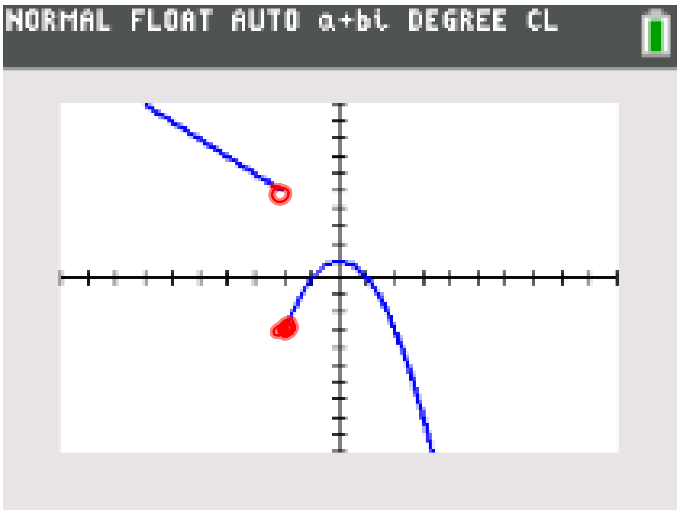
Sketch

$$f(x) = \begin{cases} 3 - x, & x < -2 \\ 1 - x^2, & x \geq -2 \end{cases}$$

$\lim_{x \to -2^-} f(x) = 5$

$\lim_{x \to -2^+} f(x) = -3$

Is -2 a point of continuity or discontinuity? If discontinuous at $x = -2$, is it removeable?



$x = -2$ is a non-removeable pt of discontinuity