2.6 Limits involving infinity -- Asymptotes

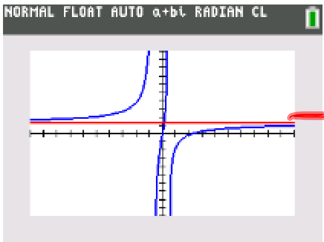
$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to-\infty} \frac{1}{x} = 0$$

this is equivalent to finding the horizontal asymptote.

Find $\displaystyle\lim_{x\to\infty} \left[\frac{4x^2-9x}{3x^2+x-1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right] =$

$$\lim_{x\to\infty} \frac{4-\frac{9}{x}}{3+\frac{1}{x}-\frac{1}{x^2}} = \frac{4-0}{3+0-0} = \frac{4}{3}$$



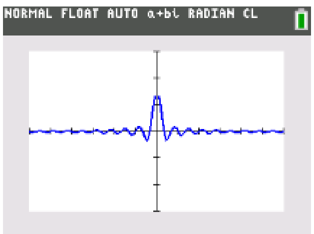
$y_2 = \frac{4}{3}$

Find $\displaystyle\lim_{x\to-\infty} \left[\frac{2x+5}{x^2-3}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right] =$

$$\lim_{x\to-\infty} \frac{\frac{2}{x}+\frac{5}{x^2}}{1-\frac{3}{x^2}} = \frac{0+0}{1-0} = \frac{0}{1} = 0$$

Find $\displaystyle\lim_{x\to\infty} \frac{\sin 3x}{2x} = 0$ $\quad\displaystyle\lim_{x\to\infty} \frac{-1}{2x} = \lim_{x\to\infty} \frac{1}{2x} = 0$

$$\frac{-1}{2x} \le \frac{\sin 3x}{2x} \le \frac{1}{2x}$$



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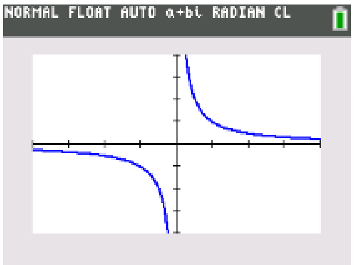
$$\lim_{t\to\infty}\left[\frac{8-8t+\sin 8t}{8t+\cos 8t}\cdot\frac{\frac{1}{t}}{\frac{1}{t}}\right]=$$

$$\lim_{t\to\infty}\frac{\frac{8}{t}-8+\frac{\sin 8t}{t}}{8+\frac{\cos 8t}{t}}=\frac{0-8+0}{8+0}=\frac{-8}{8}=-1$$

Sketch $y=\frac{1}{x}$



Note

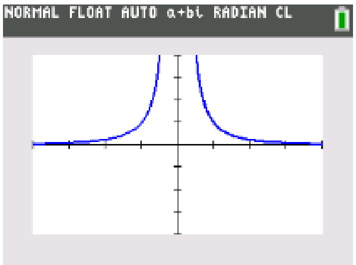
$$\lim_{x\to 0^+}\frac{1}{x}=\infty,\quad \lim_{x\to 0^-}\frac{1}{x}=-\infty$$

$\lim_{x\to 0}\frac{1}{x}$ does not exist

Sketch $y=\frac{1}{x^2}$

$$\lim_{x\to 0^+}\frac{1}{x^2}=\infty,\quad \lim_{x\to 0^-}\frac{1}{x^2}=\infty$$

$$\lim_{x\to 0}\frac{1}{x^2}=\infty$$



$$\lim_{x\to 3^-} \frac{4x}{x-3} = -\infty \qquad \frac{4(2.9)}{2.9-3}$$

$$\lim_{x\to 5^+} \frac{6-x}{x-5} = \infty \qquad \frac{6-5.1}{5.1-5}$$

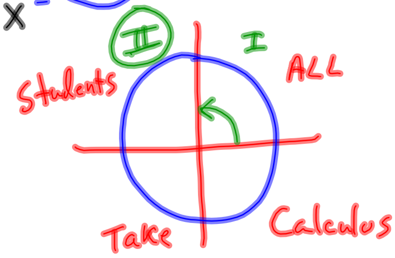
$$\lim_{x\to -2^+} \frac{-3}{x+2} = -\infty \qquad \frac{-3}{-1.9+2}$$

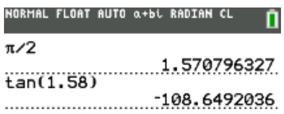
$$\lim_{x\to -4} \frac{2x}{(x+4)^2} = -\infty$$

$$\lim_{x\to -4^-} \frac{2x}{(x+4)^2} = -\infty$$

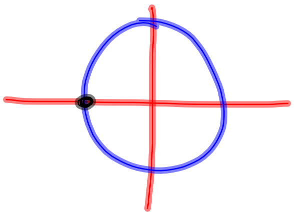
$$\lim_{x\to -4^+} \frac{2x}{(x+4)^2} = -\infty$$

$$\lim_{x\to \frac{\pi}{2}^+} \tan x = -\infty$$





$$\lim_{x\to \pi^-} \csc x = \lim_{x\to \pi^-} \frac{1}{\sin x} = \infty$$



Definition: $x=a$ is a <u>vertical asymptote</u> (VA) for $f(x)$ if $\lim_{x\to a^{+/-}} f(x) = \pm\infty$.

Ex. $f(x) = \dfrac{\sqrt{x}-9}{x-3}$; here $x=3$ is a VA.

Definition: $y=b$ is a <u>horizontal asymptote</u> (HA) if $\lim_{x\to\pm\infty} f(x) = b$.

Ex. $f(x) = \dfrac{2x+3}{5x-4}$; here $y=\frac{2}{5}$ is a HA.

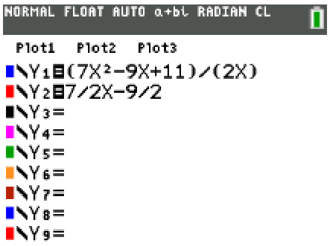
Definition: $y=mx+b$ is an oblique (OA) asymptote if $\lim_{x\to\pm\infty} f(x) = mx+b$.

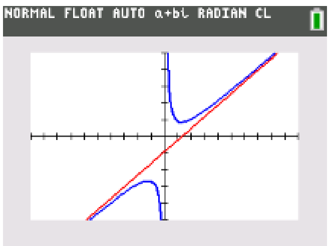
Ex. $f(x) = \dfrac{7x^2-9x+11}{2x}$

$$= \frac{7x^2}{2x} - \frac{9x}{2x} + \frac{11}{2x}$$

$$= \frac{7}{2}x - \frac{9}{2} + \underbrace{\frac{11}{2x}}_{\to 0}$$

Thus, OA: $y = \frac{7}{2}x - \frac{9}{2}$





Consider $f(x) = \frac{7x-5}{2x+7}$. Find

a) x-intercept(s): $7x-5=0 \Rightarrow x=\frac{5}{7}$ $\left(\frac{5}{7}, 0\right)$

b) y-intercept $\left(0, \frac{-5}{7}\right)$

c) VA: $2x+7=0 \Rightarrow$ $x=\frac{-7}{2}$

d) HA: $y=\frac{7}{2}$

Now consider $g(x) = \frac{x^2+3x+2}{x^2-4} = \frac{\cancel{(x+2)}(x+1)}{\cancel{(x+2)}(x-2)}$

hole in graph at $x=-2$

a) x-int: $(-1, 0)$

b) y-int: $\left(0, -\frac{1}{2}\right)$

c) VA: $x=2$

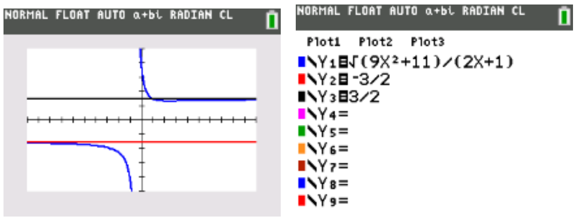
d) HA: $y=1$

Find the horizontal asymptote(s) for

$$f(x) = \frac{\sqrt{9x^2+11}}{2x+1} \qquad \sqrt{x^2} = |x|$$

$$\lim_{x\to\infty}\left[\frac{\sqrt{9x^2+11}}{2x+1}\cdot\frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{\sqrt{x^2}}}\right] = \lim_{x\to\infty}\frac{\sqrt{\frac{9x^2}{x^2}+\frac{11}{x^2}}}{\frac{2x}{|x|}+\frac{1}{|x|}} = \frac{\sqrt{9+0}}{2+0} = \frac{3}{2}$$

$$\lim_{x\to-\infty}\left[\frac{\sqrt{9x^2+11}}{2x+1}\cdot\frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{\sqrt{x^2}}}\right] = \lim_{x\to-\infty}\frac{\sqrt{\frac{9x^2}{x^2}+\frac{11}{x^2}}}{\frac{2x}{|x|}+\frac{1}{|x|}} = \frac{\sqrt{9+0}}{-2+0} = \frac{-3}{2}$$



Find the oblique asymptote for

$$f(x) = \frac{x^2-3x+4}{x} = \frac{x^2}{x} - \frac{3x}{x} + \frac{4}{x}$$

$$= x - 3 + \frac{4}{x}$$

as $x \to \pm\infty$, $f(x) \to \pm\infty$ $y = x - 3$

$$g(x) = \frac{2x^3-9x^2+4x-5}{x^2-5x+1} \qquad \frac{7x-6}{x^2-5x+1} \to 0 \text{ as } x\to\infty$$

$$\frac{2x^3}{x^2} = 2x \qquad \frac{x^2}{x^2} = 1$$

$$\begin{array}{r} 2x + 1 + \frac{7x-6}{x^2-5x+1} \\ x^2-5x+1 \overline{\big)\, 2x^3 - 9x^2 + 4x - 5} \\ 2x^3 - 10x^2 + 2x \\ \hline x^2 + 2x - 5 \\ x^2 - 5x + 1 \\ \hline 7x - 6 \end{array}$$

OA: $y = 2x+1$