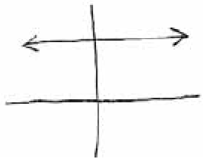
3.3 and 3.5

Differentiation Rules

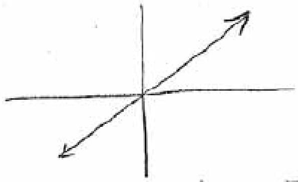
Leibnitz Notation   Newton

① $\boxed{\frac{d}{dx}(c) = 0}$ Ex. $f(x) = 3 \Rightarrow f'(x) = 0$



slope of a horizontal line is 0.

② $\boxed{\frac{d}{dx}(x) = 1}$



slope is 1

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \left[\frac{x+h-x}{h} = \frac{h}{h} = 1\right] = 1.$$

③ $\boxed{\frac{d}{dx}(x^n) = n x^{n-1}}$

proof where n is a positive integer greater than 1.

$k_{n-a} = \binom{n}{a}$

$\frac{n!}{(n-a)!\,a!}$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} =$$

$$\lim_{h\to 0} \frac{(\cancel{x^n} + k_{n-1}x^{n-1}h + k_{n-2}x^{n-2}h^2 + k_{n-3}x^{n-3}h^3 + \dots + kxh^{n-1} + h^n) - \cancel{x^n}}{h}$$

$$\lim_{h\to 0} \frac{h\left[k_{n-1}x^{n-1} + k_{n-2}x^{n-2}h + \dots h^{n-1}\right]}{h} = k_{n-1}x^{n-1}$$

$$k_{n-1} = \binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n$$

④ $\boxed{\frac{d}{dx}\left(c f(x)\right) = c f'(x)}$

⑤ $\boxed{\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) + g'(x)}$

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Find $\frac{dy}{dx}$ if

① $y = 9x^4 + 10x^2 - 4x + 7$

$\frac{dy}{dx} = 36x^3 + 20x - 4$

② $y = 5 - \frac{1}{x} + \sqrt{x} - 6\sqrt[3]{x^2} - \frac{4}{\sqrt[5]{x}}$

rewrite: $y = 5 - x^{-1} + x^{\frac{1}{2}} - 6x^{2/3} - 4x^{-1/5}$

$\frac{dy}{dx} = x^{-2} + \frac{1}{2}x^{-\frac{1}{2}} - 4x^{-\frac{1}{3}} + \frac{4}{5}x^{-6/5}$

$= \frac{1}{x^2} + \frac{1}{2\sqrt{x}} - \frac{4}{\sqrt[3]{x}} + \frac{4}{5x^{6/5}}$

③ $y = \sqrt{2x}$

$= \sqrt{2}\sqrt{x} = \sqrt{2}\, x^{\frac{1}{2}}$

OR

$\frac{dy}{dx} = \frac{\sqrt{2}}{2}x^{-\frac{1}{2}} = \frac{\sqrt{2}}{2x^{1/2}} = \frac{\sqrt{2}}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{2x}}{2x}$

For $y = 4x^3 - 6x^2 + 1$, find

a) values for x so that the tangent lines are horizontal   the slope $= 0$

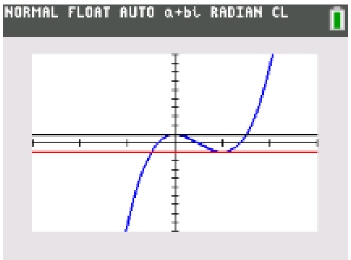
b) the equations of the horizontal lines

c) the equation of the line that contains (4, 10) and parallel to y when x=0.5

d) the equation of the line that contains (4, 10) and perpendicular to y when x=0.5

a) $\frac{dy}{dx} = 0$: $12x^2 - 12x = 0$
$12x(x-1) = 0$; $12x = 0$ | $x - 1 = 0$
$x = 0$ | $x = 1$

b) $y(0) = 1$
$y(1) = -1$
$y = 1$
$y = -1$

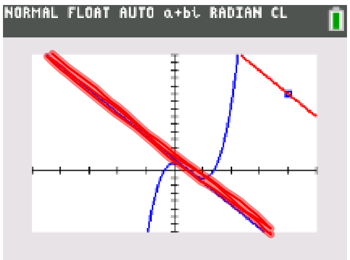


c) $\frac{dy}{dx} = 12x^2 - 12x$

$m = \frac{dy}{dx}(0.5) = 12\left(\frac{1}{2}\right)^2 - 12\left(\frac{1}{2}\right) = 3 - 6 = -3$ $(4, 10)$

$y = mx + b$; $10 = -3(4) + b$
$10 = -12 + b \Rightarrow b = 22$

$y = -3x + 22$



d) $m = -3$ $m\, m_\perp = -1$
$m_\perp = \frac{1}{3}$

$y = mx + b$; $10 = \frac{1}{3}(4) + b$
$10 = \frac{4}{3} + b \Rightarrow b = 10 - \frac{4}{3}$
$= \frac{30}{3} - \frac{4}{3} = \frac{26}{3}$

$y = \frac{1}{3}x + \frac{26}{3}$

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(6) $\boxed{\frac{d}{dx}(\sin x) = \cos x}$

$\sin(u+v) = \sin u \cos v + \sin v \cos u$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} =$$

factor out $\sin x$

$$\lim_{h \to 0} \left[\frac{\sin x \cosh + \sinh \cos x - \sin x}{h} = \frac{\sin x \cosh - \sin x + \sinh \cos x}{h}\right]$$

$$\lim_{h \to 0} \left[\frac{\sin x(\cosh - 1) + \sinh \cos x}{h} = \frac{\cosh - 1}{h}\sin x + \frac{\sinh}{h}\cos x\right] =$$

$$\sin x \lim_{h \to 0} \frac{\cosh - 1}{h} + \cos x \lim_{h \to 0} \frac{\sinh}{h} = (\sin x)(0) + \cos x(1) = \cos x.$$

(7) $\boxed{\frac{d}{dx}(\cos x) = -\sin x}$

Product Rule

If $F(x) = f(x)g(x)$ then
$F'(x) = f(x)g'(x) + g(x)f'(x)$

i.e. $\frac{d}{dx}(uv) = uv' + vu'$

Example: find $\frac{dy}{dx}$ if $y = \underbrace{(2x+5)}_{u}\underbrace{(3x-2)}_{v}$ using product then verify result using foil.

$\frac{dy}{dx} = (2x+5)(3) + (3x-2)(2)$
$= 6x + 15 + 6x - 4 = 12x + 11$

rewrite y: $y = 6x^2 + 11x - 10$
$\frac{dy}{dx} = 12x + 11$

Ex. Find $f'(x)$ if $f(x) = \underbrace{x \cdot \sin x}_{u_1 v_1} + \underbrace{3x \cdot \cos x}_{u_2 v_2}$

$f'(x) = x\cos x + \sin x(1) + 3x(-\sin x) + \cos x(3)$

$= x\cos x + \sin x - 3x\sin x + 3\cos x$

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Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

Ex. $f(x) = \dfrac{2x-3}{x+1}$

$$f'(x) = \frac{(x+1)(2) - (2x-3)(1)}{(x+1)^2} = \frac{2x+2-2x+3}{(x+1)^2}$$

$$= \frac{5}{(x+1)^2}$$

Find $f'(x)$ if $f(x) = \dfrac{3x^4 - 7x^2 + 1}{x}$

AVOID QUOTIENT RULE IF POSSIBLE

$$f(x) = \frac{3x^4}{x} - \frac{7x^2}{x} + \frac{1}{x}$$

$$= 3x^3 - 7x + x^{-1}$$

$$f'(x) = 9x^2 - 7 - x^{-2} \quad \text{or} \quad 9x^2 - 7 - \frac{1}{x^2}$$

Ex. $f(x) = \tan x = \dfrac{\sin x}{\cos x}$

$$f'(x) = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

Trig derivatives

① $\frac{d}{dx}(\sin x) = \cos x$

② $\frac{d}{dx}(\cos x) = -\sin x$

③ $\frac{d}{dx}(\tan x) = \sec^2 x$

④ $\frac{d}{dx}(\cot x) = -\csc^2 x$

⑤ $\frac{d}{dx}(\sec x) = \sec x \tan x$

⑥ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

Ex. $y = \frac{1 - \cos x}{\sin x}$

AVOID QR

$$y = \frac{1}{\sin x} - \frac{\cos x}{\sin x}$$

$$= \csc x - \cot x$$

$$\frac{dy}{dx} = -\csc x \cot x - \overset{\oplus}{(-\csc^2 x)}$$

$$= \csc^2 x - \csc x \cot x$$

$$= \csc x(\csc x - \cot x)$$

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Notation:

1st derivative: y', $f'(x)$, $\frac{dy}{dx}$

2nd " : y'', $f''(x)$, $\frac{d^2y}{dx^2}$

3th " : y''', $f'''(x)$, $\frac{d^3y}{dx^3}$

4th " : $y^{(4)}$, $f^{(4)}(x)$, $\frac{d^4y}{dx^4}$

Find all derivatives for $f(x) = 4x^3$

$f'(x) = 12x^2$

$f''(x) = 24x$

$f'''(x) = 24$

$f^{(4)}(x) = 0$

$f^{(5)}(x) = 0$

$\vdots$

$f^{(\infty)}(x) = 0$

Thus, if the distance traveled by an object is $s(t) = 0.04t^2 + t$ feet after t second

Its velocity is $v(t) = s'(t) = 0.08t + 1$

and

Its acceleration is $a(t) = v'(t) = s''(t) = 0.08$.

Its jerk is $j(t) = a'(t) = v''(t) = s'''(t)$

$j(t) = 0.$

A rocket is shoot straight up from a 100-foot building. Its distance above the ground after t seconds is $s(t) = -16t^2 + 400t + 100$.

a) what is its velocity when $t = 1$, $t = 8$, $t = 12.5$, $t = 15$?

$v(t) = s'(t) = -32t + 400$

$v(1) = 368$ ft/sec

$v(8) = 144$ ft/sec

$v(12.5) = 0$ ft/sec

$v(15) = -32(15) + 400 = -80$ ft/sec

NORMAL FLOAT AUTO a+bi RADIAN CL

-16(12.5)²+400(12.5)+100
2600

b) Max Height? $s(12.5) = 2600$ ft

c) Is the acceleration constant? $v(t) = -32t + 400$

$a(t) = v'(t) = s''(t) = -32$ ft/sec²

YES