

3.4 The Derivative as Rate of Change

The position of an ant with respect to a lump of substitute sugar in inches after t seconds is given by

$$x(t) = t^2 - 9t + 8, 0 \leq t \leq 8$$

- Find the displacement of the ant and the ant's average velocity for the given time interval.
- Find the ant's speed and acceleration at the endpoints of the time interval.
- When, if ever, during the time interval does the ant change direction?

$$\begin{aligned} \text{a) } \Delta(x) - \Delta(0) &= (64 - 72 + 8) - (0 - 0 + 8) \\ &= 0 - 8 = -8 \text{ inches} \end{aligned}$$

$$v = \frac{\Delta \Delta}{\Delta t} = \frac{-8}{8} = -1 \text{ in/sec}$$

$$\text{b) } v(t) = 2t - 9$$

$$a(t) = 2 \text{ in/sec}^2 \text{ everywhere}$$

$$v(0) = -9 \text{ in/sec}$$

$$v(8) = 16 - 9 = 7 \text{ in/sec}$$

$$\text{Speed: } 9 \text{ in/sec}$$

$$\text{speed: } 7 \text{ in/sec}$$

$$\text{c) When is } v(t) = 0?$$

$$2t - 9 = 0 \Rightarrow t = 4.5 \text{ sec}$$

The position of a snail with respect to a leaf in inches after t minutes is given by $x(t) = t^3 - 9t^2 + 24t, 0 \leq t \leq 5$.

- Find the snail's acceleration each time the velocity is zero.
- Find the snail's speed each time its acceleration is zero.
- Find the total distance traveled by the body from $t=0$ to $t=3$.

$$a) \quad v(t) = x'(t) = 3t^2 - 18t + 24$$

$$\text{Set } 3t^2 - 18t + 24 = 0$$

$$t^2 - 6t + 8 = 0$$

$$(t-2)(t-4) = 0 \Rightarrow t = 2, t = 4$$

$$a(t) = x''(t) = 6t - 18$$

$$a(2) = -6$$

$$a(4) = 6$$

$$b) \quad 6t - 18 = 0 \Rightarrow t = 3$$

$$v(3) = 3(3)^2 - 18(3) + 24 = 27 - 54 + 24 = -3$$

$$c) \quad \Delta(2) = 20$$

$$\frac{20}{2} = 22 \text{ inches}$$



NORMAL FLOAT AUTO a+bL RADIAN CL

X1r(2) 20

X1r(3)-X1r(2) -2