3.6 Chain Rule

$$\frac{dy}{dx} = \frac{dy}{\cancel{du}} \cdot \frac{\cancel{du}}{dx}$$ quasi cancellation

u(x) is often called the inside function- the "stuff" in parentheses.

Example: Let $y = \sin(3x^4 - 7)$

Find $\frac{dy}{dx}$:

let $u = 3x^4 - 7$
$y = \sin u$

(you)(x)

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos u \cdot (12x^3)$$
$$= \cos(3x^4 - 7)(12x^3)$$
$$= 12x^3 \cos(3x^4 - 7)$$

Alternative Form: $\frac{d}{dx}\{f[g(x)]\} = \underline{f'[g(x)]} \cdot g'(x)$

Example: $f(x) = \cos(1 - 2x)$

$$= -\sin(1-2x) \cdot (-2)$$
$$= 2\sin(1-2x)$$

Example: $f(x) = \sin(7x^4 - 5x + 2)$

$$f'(x) = (28x^3 - 5)\cos(7x^4 - 5x + 2)$$

Example: $y = (3x^2 - 6x + 4)^8$

Let $u = 3x^2 - 6x + 4$

$y = u^8$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 8u^7 \cdot (6x - 6)$$

$$= 8(3x^2 - 6x + 4)^7 [6(x-1)]$$

$$= 48(x-1)(3x^2 - 6x + 4)^7$$

Generalized Power Rule:

$$\frac{d}{dx}\left\{[f(x)]^n\right\} = n[f(x)]^{n-1} \cdot f'(x)$$

Example: $f(x) = (3x^4 - 5x + 7)^{96}$

$$f'(x) = 96(3x^4 - 5x + 7)^{95}(12x^3 - 5)$$

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Algebra Review

1. Simplify $2x^3 + 10x^2 - 8x =$

$2x(x^2 + 5x - 4)$

2. Simplify $4x^{-2} + 8x^2 - 10x^{-1}$

$2x^{-2}(2 + 4x^4 - 5x)$

3. Simplify $8\sqrt{x} - \frac{4}{\sqrt{x}}$

$8x^{\frac{1}{2}} - 4x^{-\frac{1}{2}} =$

$4x^{-\frac{1}{2}}(2x - 1)$

*Example:

$$g(x) = 2x^2\sqrt{x^2+1}$$
$$= \underset{u}{2x^2}\cdot\underset{v}{(x^2+1)^{\frac{1}{2}}}$$

$\frac{d}{dx}(u \cdot v) = uv' + vu'$

$$g'(x) = 2x^2\left[\tfrac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)\right] + (x^2+1)^{\frac{1}{2}}(4x)$$
$$= 2x^3(x^2+1)^{-\frac{1}{2}} + 4x(x^2+1)^{\frac{1}{2}}$$
$$= 2x(x^2+1)^{-\frac{1}{2}}\left(x^2 + 2(x^2+1)^1\right)$$
$$= 2x(x^2+1)^{-\frac{1}{2}}(3x^2+2) \quad \text{OR} \quad = \frac{2x(3x^2+2)}{\sqrt{x^2+1}}$$

Example:

$$y = \sqrt[3]{(7x^2-x)^2} \quad ; \quad y = (7x^2-x)^{2/3}$$

$$\frac{dy}{dx} = \frac{2}{3}(7x^2-x)^{-\frac{1}{3}}(14x-1) \quad \text{OR} \quad = \frac{2(14x-1)}{3\sqrt[3]{7x^2-x}}$$

Example: $f(x) = \sin\left[x(x+1)^{-1}\right]$

$$f(x) = \sin\left(\frac{x}{x+1}\right)$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

$$f'(x) = \cos\left(\frac{x}{x+1}\right)\cdot\frac{d}{dx}\left(\frac{x}{x+1}\right)$$

$$= \cos\left(\frac{x}{x+1}\right)\cdot\frac{(x+1)(1) - x(1)}{(x+1)^2} = \cos\left(\frac{x}{x+1}\right)\cdot\frac{1}{(x+1)^2}$$

$$= \frac{\cos\left(\frac{x}{x+1}\right)}{(x+1)^2}$$

Nested: inside function inside an inside function

Example: $f(x) = \cos^3(4x^5 - x)$

Rewrite $f(x) = \left[\cos(4x^5 - x)\right]^3$

$$f'(x) = 3\left[\cos(4x^5 - x)\right]^2\cdot\frac{d}{dx}\left[\cos(4x^5 - x)\right]$$

$$= 3\cos^2(4x^5 - x)\cdot\left[-\sin(4x^5 - x)\right]\cdot\frac{d}{dx}(4x^5 - x)$$

$$= 3\cos^2(4x^5 - x)\left[-\sin(4x^5 - x)\right]\cdot(20x^4 - 1)$$

$$= -3(20x^4 - 1)\cos^2(4x^5 - x)\sin(4x^5 - x)$$

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Example: $f(x) = \sin^7\left(\sqrt{x^2-x}\right)$

Rewrite $f(x) = \left(\sin\sqrt{x^2-x}\right)^7$

$$f'(x) = 7\left(\sin\sqrt{x^2-x}\right)^6 \cdot \cos\sqrt{x^2-x} \cdot \frac{d}{dx}(x^2-x)^{\frac{1}{2}}$$

$$= 7\sin^6\sqrt{x^2-x} \cdot \cos\sqrt{x^2-x} \cdot \frac{1}{2}(x^2-x)^{-\frac{1}{2}}(2x-1)$$

$$= \frac{7(2x-1)\sin^6\sqrt{x^2-x}\,\cos\sqrt{x^2-x}}{2\sqrt{x^2-x}}$$

Example: $f(x) = \tan^5(5x^3)$

$$f(x) = \left[\tan(5x^3)\right]^5$$

$$f'(x) = 5\left[\tan(5x^3)\right]^4 \cdot \sec^2(5x^3) \cdot (15x^2)$$

$$= 75x^2\tan^4(5x^3) \cdot \sec^2(5x^3)$$

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Example: $g(x) = \sec^3(3x^2-7)$

$$g(x) = \left[\sec(3x^2-7)\right]^3$$

$$g'(x) = 3\boxed{\sec(3x^2-7)}^2 \cdot \sec(3x^2-7)\tan(3x^2-7)\cdot(6x)$$

$$= 18x\sec^2(3x^2-7)\sec(3x^2-7)\tan(3x^2-7)$$

$$= 18x\sec^3(3x^2-7)\tan(3x^2-7)$$

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Example: $y = \sin^{10}(\cos 3x)$

$$y = [\sin(\cos 3x)]^{10}$$

$$\frac{dy}{dx} = 10[\sin(\cos 3x)]^9 \cdot \frac{d}{dx}[\sin(\cos 3x)]$$

$$= 10 \sin^9(\cos 3x) \cdot \cos(\cos 3x) \cdot \frac{d}{dx}(\cos 3x)$$

$$= 10 \sin^9(\cos 3x) \cdot \cos(\cos 3x) \cdot [-\sin(3x)] \cdot 3$$

$$= -30 \sin^9(\cos 3x) \cos(\cos 3x) \sin 3x$$

You try one:

$$y = \cos^3(3x-1)$$

$$= [\cos(3x-1)]^3$$

$$\frac{dy}{dx} = 3[\cos(3x-1)]^2 [-\sin(3x-1)] \cdot 3$$

$$= -9\cos^2(3x-1)\sin(3x-1)$$

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