3.7 Implicit Differentiation

Implicit means hidden. Here, the explicit solution for the dependent variable (typically y) will not be needed. This technique works best for curves that are not functions.

Example: Find the slope of the tangent line to the curve

$x^2+y^2 = 25$ at (3, 4).

Solution not using implicit differentiation.

Solve for y

$$y^2 = 25 - x^2 \Rightarrow y = \pm\sqrt{25-x^2}$$
$$= \pm(25-x^2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \pm\frac{1}{2}(25-x^2)^{-\frac{1}{2}}(-2x)$$
$$= \frac{\pm x}{\sqrt{25-x^2}}; \text{ at } x=3,$$

the slope is either $\frac{3}{4}$ or $\frac{-3}{4}$.

When done correctly, $\frac{dy}{dx}$ is read as the derivative of the function y with respect to the variable x. y is an inside function

Redo problem implicitly.

$$\frac{d}{dx}\left(x^2+(y)^2\right)=\frac{d}{dx}(25) \Rightarrow$$

$$2x+2y\frac{dy}{dx}=0 \quad \Rightarrow 2y\frac{dy}{dx}=-2x$$

$$\frac{dy}{dx}=\frac{-2x}{2y}=\frac{-x}{y}$$

Recall,
$y=\pm\sqrt{25-x^2}$
a function of x. So,
$y^2=\left(\pm\sqrt{25-x^2}\right)^2$
$\frac{d}{dx}(y^2)=$
$2y\cdot\frac{dy}{dx}$

Thus, the slope of the tangent line at (3,4) is $\frac{-3}{4}$. No ambuguity.

Find the ~~equation~~ slope of the tangent line at (2,4) for the curve

$x^3 + y^3 = 9xy$ [Folium of Descartes]

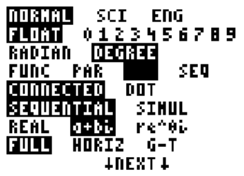
$x = r\cos\theta$

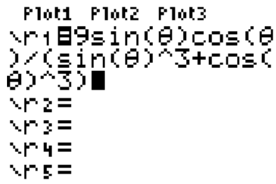
$y = r\sin\theta$

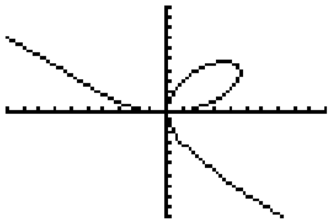
$x^2 + y^2 = r^2$

polar form

$$r = \frac{9\sin\theta\cos\theta}{\cos^3\theta + \sin^3\theta}$$

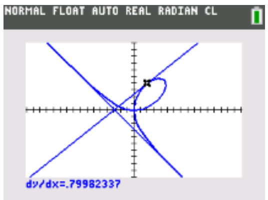






at $(2,4)$; $r^2 = 2^2 + 4^2 = 20$

$2 = \sqrt{20}\cos\theta \Rightarrow \cos\theta = \frac{2}{\sqrt{20}} \Rightarrow \theta = 63.435°$
$= 1.1071$ rad.



$$\frac{d}{dx}(uv) = uv' + vu'$$

$x^3 + y^3 = 9xy$; $\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(\underbrace{9x}_{u}\cdot\underbrace{y}_{v})$

$$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 9x \cdot \frac{dy}{dx} + y \cdot 9$$

$$3x^2 + 3y^2\frac{dy}{dx} = 9x\frac{dy}{dx} + 9y$$

$$3y^2\frac{dy}{dx} - 9x\frac{dy}{dx} = 9y - 3x^2$$

$$y^2\frac{dy}{dx} - 3x\frac{dy}{dx} = 3y - x^2$$

$$\frac{dy}{dx}(y^2 - 3x) = 3y - x^2$$

$$\frac{dy}{dx} = \frac{3y - x^2}{y^2 - 3x}\ ;\quad m = \left.\frac{dy}{dx}\right|_{(2,4)} = \frac{3(4) - 2^2}{4^2 - 3(2)}$$

$$= \frac{12 - 4}{16 - 6} = \frac{8}{10} = .8$$