3.8 Related Rates: $\frac{d}{dt}$

Say x and y are changing with respect to time. That is, we could write x(t) and y (t).

Also, we have the following relationship:

$$8y^3 + x^2 = 1$$

Find $\frac{dy}{dt}$ when $\frac{dx}{dt} = 2$

$x = 3$

$y = -1$

$x(t) = ?$

$y(t) = ?$

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$$\frac{d}{dt}\left(8y^3 + x^2\right) = \frac{d}{dt}(1)$$

$$24y^2 \cdot \frac{dy}{dt} + 2x \cdot \frac{dx}{dt} = 0$$

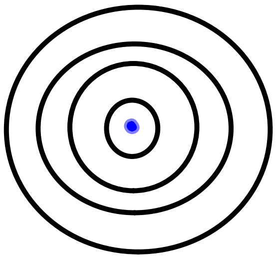
$$24(-1)^2 \cdot \frac{dy}{dt} + 2(3)\cdot(2) = 0$$

$$24\frac{dy}{dt} + 12 = 0$$

$$\frac{dy}{dt} = \frac{-12}{24} = -\frac{1}{2}$$

Jun 9-1:40 PM

A pebble is dropped into a pond and concentric circles form. If the radius is increasing at 8 inches per second, how fast is the area of the outer circle changing when the radius of the outer circle is 60 inches?



$A = \pi r^2$

$\frac{d}{dt} A = \frac{d}{dt}(\pi r^2)$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$= 2\pi (60\text{ in}) \cdot \left(\frac{8\text{ in}}{\text{sec}}\right) = 960\pi \text{ in}^2/\text{sec}$

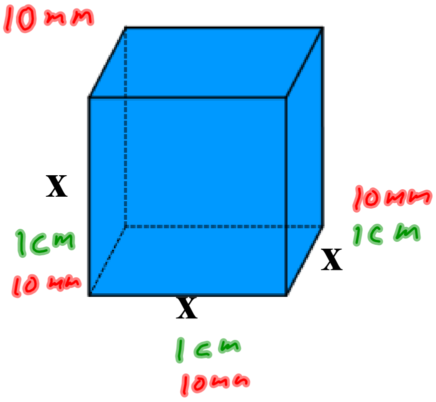
$\frac{dr}{dt} = 8$ in/sec

$r = 60$ in

$\frac{dA}{dt} = ?$

Jun 4-8:35 AM

All edges of an ice cube are melting at a rate of 5mm per minute. How fast is the volume of the cube changing when the lengths of the edges are 1cm? = 10 mm



$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$\frac{dx}{dt} = -5 \text{ mm/min}$

$x = 10 \text{ mm}$

$\frac{dV}{dt} = ?$

$= 3(10\text{mm})^2 \cdot (-5 \text{ mm/min})$

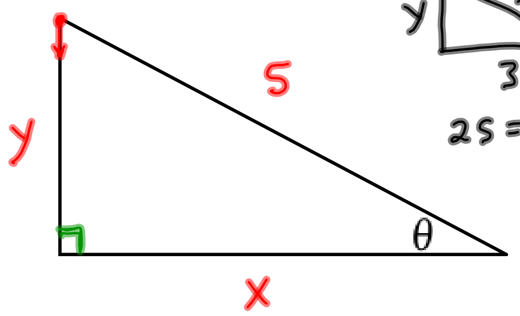
$= -1500 \text{ mm}^3/\text{min} = \underline{-1.5} \text{ cm}^3/\text{min}$

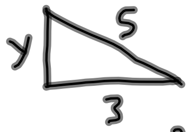
$\frac{-1500 \cancel{\text{mm}^3}}{\text{min}} \cdot \frac{1\text{cm}}{10 \cancel{\text{mm}}} \cdot \frac{1\text{cm}}{10 \cancel{\text{mm}}} \cdot \frac{1\text{cm}}{10 \cancel{\text{mm}}}$

A 5-meter ladder resting on a wall begins to slip. The top of the ladder is falling at a rate of 1.5 m/sec when the bottom of the ladder is 3 meters from the wall.

At this time, a) how fast is the bottom of the ladder moving away from the wall? b) How fast is the area of the triangle changing?

c) How fast is the angle that the ladder makes with the wall changing?





$25 = y^2 + 3^2$

$y^2 = 16$

$y = 4$

$\frac{dy}{dt} = -1.5 \text{ m/sec}$

$x = 3$

$\frac{dx}{dt} = ?$

a) $x^2 + y^2 = 5^2 \Longrightarrow \frac{d}{dt}(25) = 0$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \quad \div 2$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$(3)\frac{dx}{dt} + (4)(-1.5) = 0$

$3\frac{dx}{dt} = 6 \Rightarrow \frac{dx}{dt} = 2 \text{ m/sec}$

Jun 4-8:36 AM

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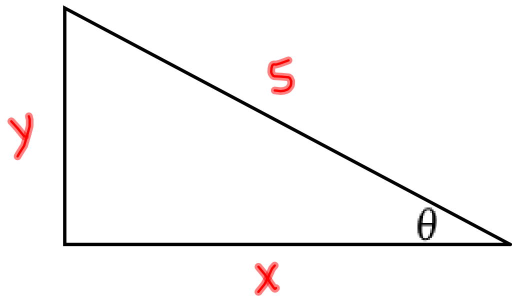

$\frac{dy}{dt} = -1.5$

$\frac{dx}{dt} = 2$

$x = 3, \ y = 4$

$\boxed{A = \frac{1}{2} b h}$ formula

$\frac{d}{dx}(c f(x)) = c f'(x)$

$A = \frac{1}{2} x \cdot y = \frac{1}{2}(x \cdot y)$

$\frac{dA}{dt} = \frac{1}{2}\left(x \frac{dy}{dt} + y \cdot \frac{dx}{dt}\right)$

$= \frac{1}{2}\left[(3) \cdot (-1.5) + (4)(2)\right]$

$= \frac{1}{2}(-4.5 + 8) = \frac{1}{2}(3.5) = 1.75 \text{ m}^2/\text{sec}$

Jun 4-8:36 AM

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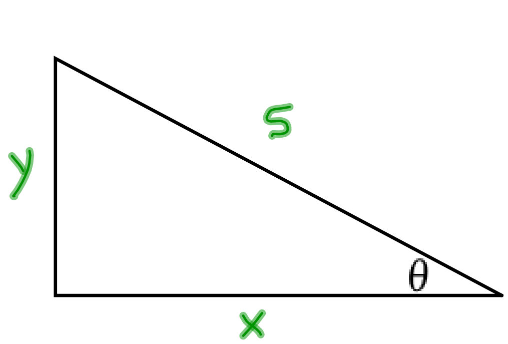
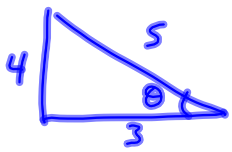


$$\sin\theta = \frac{y}{5}$$

$$\sin(\theta) = \frac{1}{5}y$$

$$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{5}\frac{dy}{dt}$$



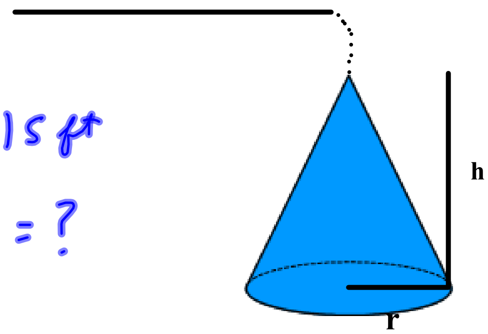
$$\cos\theta = \frac{3}{5}$$

$$(.6)\frac{d\theta}{dt} = (.2)(-1.5)$$

$$\frac{d\theta}{dt} = \frac{(.2)(-1.5)}{.6} = \frac{-.5\text{ rad}}{\text{sec}} \approx -28.6^\circ/\text{sec}$$

Jun 4-8:36 AM

At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base is 3 times its height. At what rate is the height changing when it is 15 feet high?



$h = 15\text{ ft}$

$\frac{dh}{dt} = ?$

$\frac{dV}{dt} = 10\text{ ft}^3/\text{min}$

$d = 3h$

$2r = 3h$

$r = \frac{3}{2}h$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{3}{2}h\right)^2 h$$

$$= \frac{1}{3}\pi\left(\frac{9}{4}h^2\right)h$$

$$V = \frac{3}{4}\pi h^3$$

$$\frac{dV}{dt} = \frac{9}{4}\pi h^2 \frac{dh}{dt}$$

$$10 = \frac{9}{4}\pi (15)^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{10}{2.25\pi(225)}$$

$$\approx 0.006\text{ ft/min}$$

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NORMAL FLOAT AUTO REAL RADIAN CL
2.25*225
                          506.25
Ans*π
                     1590.431281
10/Ans
                     .0062876027
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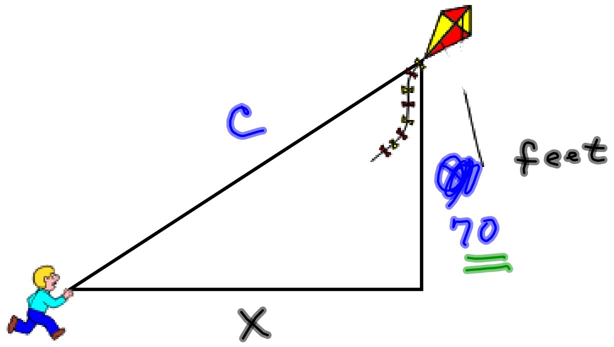
Ben Franklin is flying a kite where the height of the kite is 70 feet above the spool of string. The wind carries the kite horizontally away from Ben at a rate of 25 ft/sec. How fast must Ben let out the string when the kite is 250 ft from him? (Kite stays at 70 feet in height)



$y = 70$

$\frac{dx}{dt} = 25$

$\frac{dc}{dt} = ?$

$c = 250$

$250^2 = x^2 + 70^2$

NORMAL FLOAT AUTO REAL RADIAN CL
250²−70²
57600
√(Ans)
240

$c^2 = x^2 + 70^2$

$2c\frac{dc}{dt} = 2x\frac{dx}{dt}$

$c\frac{dc}{dt} = x\frac{dx}{dt}$

$(250)\frac{dc}{dt} = (240)(25)$

NORMAL FLOAT AUTO REAL RADIAN CL
240*25/250
24

$\frac{dc}{dt} = 24 \text{ ft/sec}$

Jun 4-8:43 AM

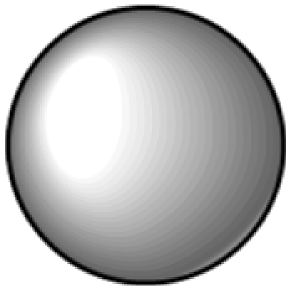
A spherical balloon is being filled with air at a rate of 500 cubic feet per minute. a) How fast is the radius changing when the radius is 4 feet? b) How fast is the surface area changing?

a) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 500$

$\frac{dr}{dt} = ?$

$r = 4$



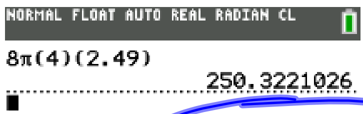
$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$500 = 4\pi(4)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{500}{64\pi} \approx 2.49$ ft/min

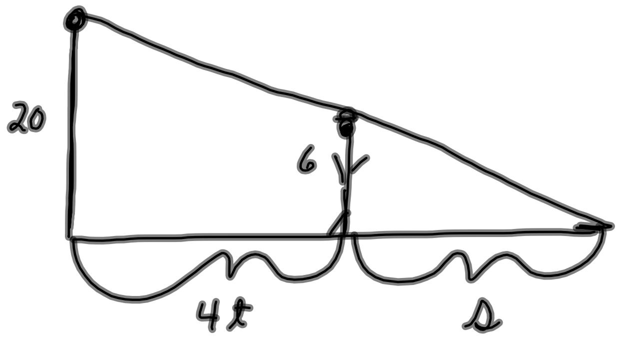
b) $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$= 8\pi(4)(2.49)$



≈ 250 ft^2/min

A 20-foot lamp is casting a shadow on a 6- foot man walking away at 4 feet per second. How fast is the length of the shadow changing?



$d = rt$

$d = 4t$

$\frac{ds}{dt} = ?$

$$\frac{20}{4t+s} = \frac{6}{s} \Rightarrow 20s = 24t + 6s$$

$$14s = 24t$$

$$7s = 12t$$

$$7\frac{ds}{dt} = 12$$

$$\frac{ds}{dt} = \frac{12}{7} \text{ ft/sec}$$

Jun 10-3:31 PM