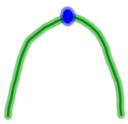
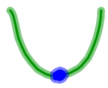
4.1 **Extrema** (plural of Extremum)

Extremum (collective term for **maximum, minimum**)



extremum is a max



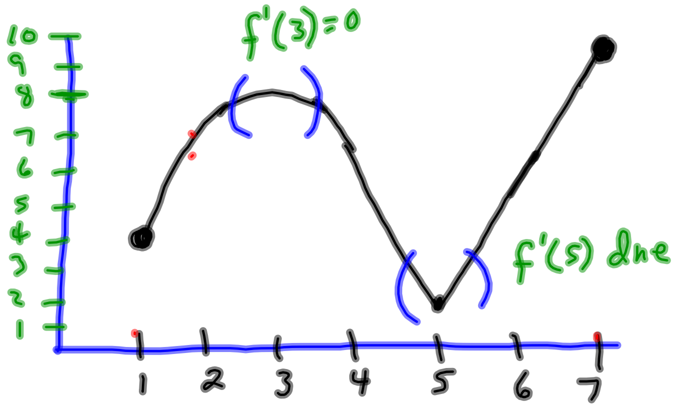
extremum is a min

plural is extrema, minima, maxima

Local extremum (relative extremum) versus extremum (absolute extremum)

Local Extremum:

If there exists an open interval (a,b) for which c is in (a,b) and f (c) is the largest or smallest value for every x in (a, b) then f(c) is a **<u>relative / local extremum</u>**.



local max at $x=3$
local max is $f(3)=8$
$(3,8)$

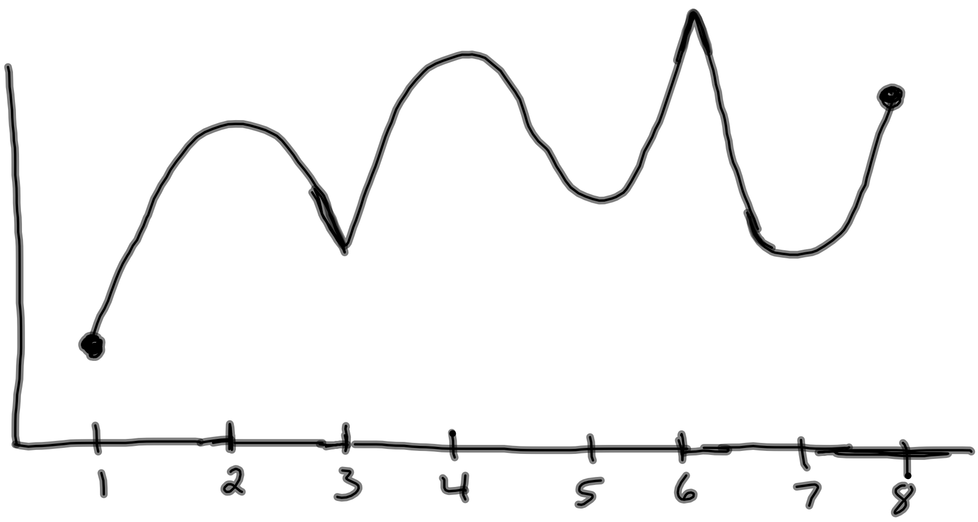
local min at $x=5$
local min is $f(5)=2$
$(5,2)$

absolute min is $(5,2)$
absolute max is $(7,10)$

We are looking for hills, valleys, peaks

Absolute Extremum:

If f(c) is the largest or smallest value for every x in (a,b) or [a,b] containing c then f(c) is an **absolute extremum for the interval**.



① Where is $f'(x)=0$? $x = 2, 4, 5, 7$

② Where is $f'(x)$ undefined? $x = 3, 6$

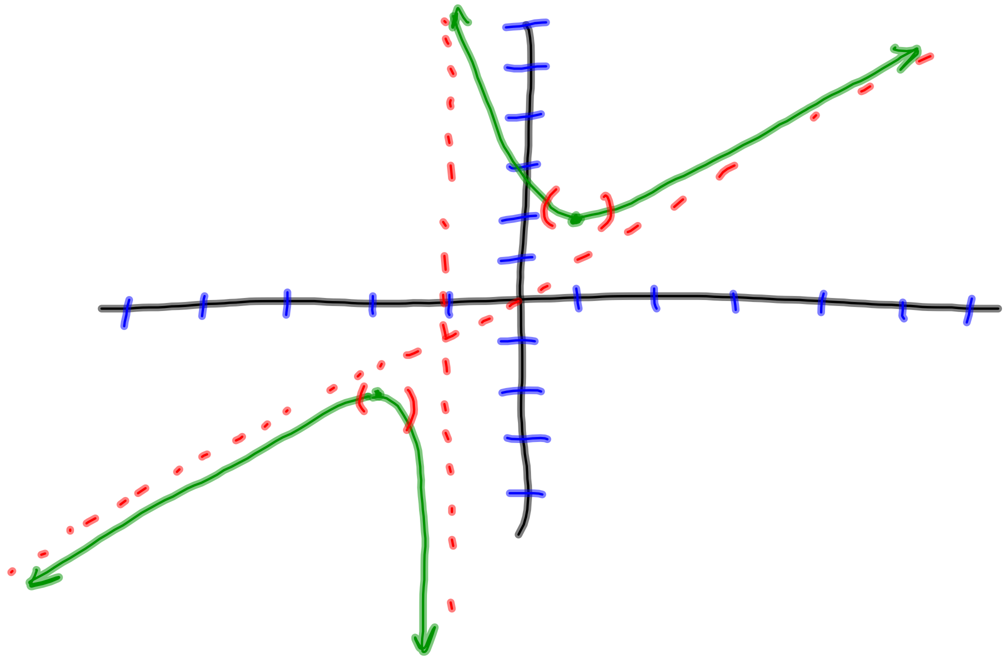
③ Where are the local mins? $x = 3, 5, 7$

④ Where are the local maxs? $x = 2, 4, 6$

⑤ Where is the min? $x = 1$

⑥ Where is the max? $x = 6$

Jun 10-1:02 PM



Identify local extrema and (absolute) extrema

local min (1, 2)
local max (-2, -2)

abs max is kinda ∞ : no abs max since ∞ is not a number

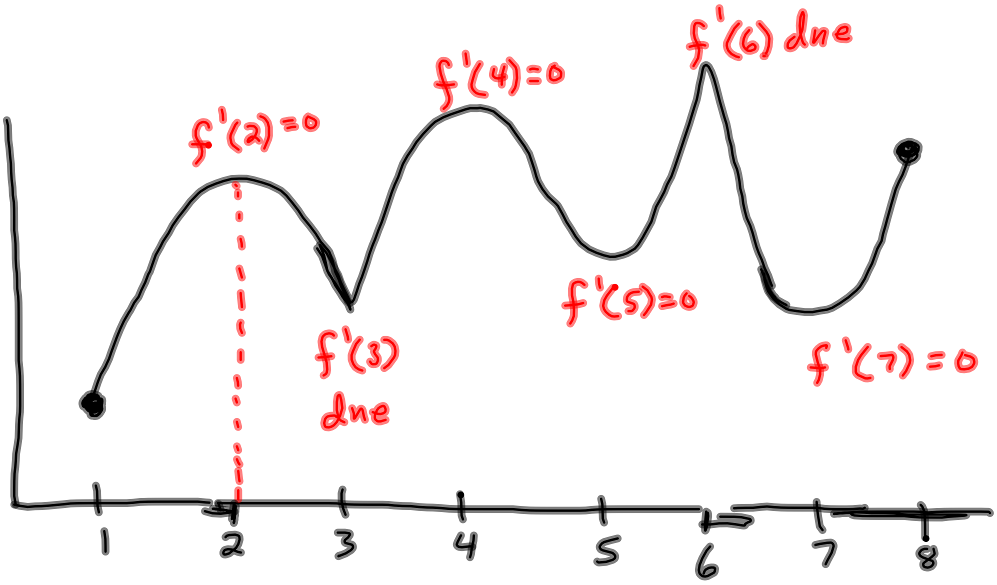
abs min is kind -∞ : no abs min since -∞ is not a number

Jun 16-11:17 AM

Definition: If $f(c)$ exists and $f'(c) = 0$ (hills, valleys)
or
$f'(c)$ does not exist, then (peaks)
c is called a critical value. (cv)

Theorem: If $f(c)$ is a local extremum then c is a critical value.

Theorem: If $f(c)$ is an absolute extremum on $[a, b]$ then c is a critical value or endpoint.



local max at $x = 2, 4, 6$
local min at $x = 3, 5, 7$
max at $x = 6$ (also a local max)
min at $x = 1$ (not a local min, end point)

Ex. Find the absolute extrema

for $f(x) = 6(x^2-2x-3)^{2/3}$

on a) $[-1.5, 4]$ b) $(-\infty, \infty)$

a) $f'(x) = \frac{2}{3}(6)(x^2-2x-3)^{-\frac{1}{3}}(2x-2)$

$= 4(x^2-2x-3)^{-1/3}(2)(x-1)$

$= \dfrac{8(x-1)}{\sqrt[3]{x^2-2x-3}} = \dfrac{8(x-1)}{\sqrt[3]{(x-3)(x+1)}}$

$f'(x) = 0$ when $x = 1$

$f'(x)$ dne when $x = -1, 3$

note: $f(-1)$ exists, $f(3)$ exists

CV: $\underline{-1, 3}$, 1

probably sharp points

To find absolute extrema, test CV's & endpoints

$f(-1) =$

$f(1) =$

$f(3) =$

$f(-1.5) =$

$f(4) =$

NORMAL FLOAT AUTO REAL RADIAN CL

X	Y1
-1	0
1	15.119
3	0
-1.5	10.302
4	17.544

X=

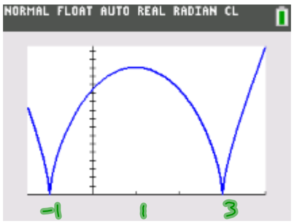
abs min when $x = -1, 3$

abs max when $x = 4$

Zoom fit (0)

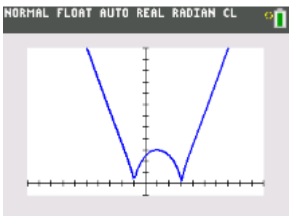
NORMAL FLOAT AUTO REAL RADIAN CL

WINDOW
Xmin=-1.5
Xmax=4
Xscl=1
Ymin=0 abs min
Ymax=17.54410643 abs max
Yscl=1
Xres=1
ΔX=.0208333333333
TraceStep=.04166666666666



exact abs max : $f(4) = 6(5)^{2/3} = 6(25)^{1/3} = 6\sqrt[3]{25}$

(b)



abs min is $(-1,0)$, $(3,0)$

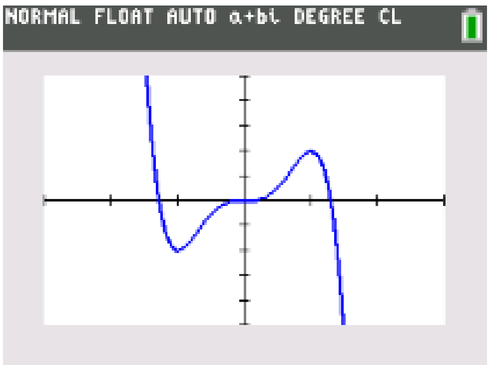
no abs max

Note: If c is a CV then that $f(c)$ is a relative does not imply extremum.

Ex. Find the relative extrema and absolute extrema for $f(x) = -3x^5 + 5x^3$ on $(-\infty, \infty)$.

$$f'(x) = -15x^4 + 15x^2$$
$$= -15x^2(x^2-1) = -15x^2(x+1)(x-1)$$

CV's: 0, -1, 1



rel min at $x = -1$
$f(-1) = -2$

rel max at $x = 1$
$f(1) = 2$

b) Since it's unbounded above and below the absolute extrema do not exist.

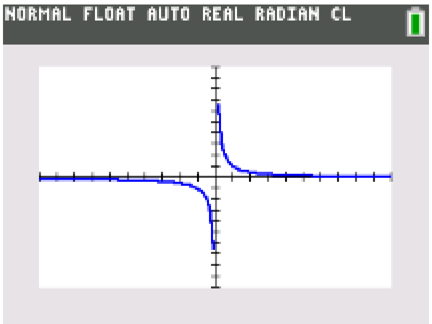
Jun 10-1:34 PM

Ex. Find the relative extrema for

$f(x) = \frac{1}{x}$; Neither $f(0)$ nor $f'(0)$ exists !!!

$f(x) = x^{-1} \Rightarrow f'(x) = -x^{-2} = \frac{-1}{x^2}$

CV's: None No relative extrema



Ex. Find the relative and absolute extrema for

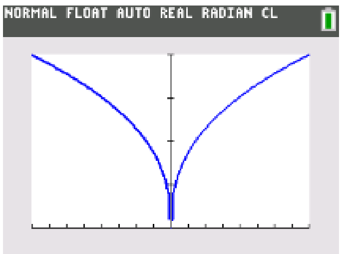
$f(x) = x^{2/5}$ on a) $(-32, 32)$

b) $[-32, 32]$

$f(0)$ exists, $f(0) = 0$

a) $f'(x) = \frac{2}{5}x^{-3/5} = \frac{2}{5x^{3/5}}$ $f'(0)$ dne

CV: 0



abs min at $(0,0)$ on $(-32, 32)$

no max on $(-32, 32)$

b) abs min at $(0,0)$ on $[-32, 32]$

abs max at $(32, 4)$, $(-32, 4)$ on $[-32, 32]$

Find <u>absolute</u> extrema for $f(x) = 8x - x^4$ on $[-1, 2]$.

$f'(x) = 8 - 4x^3$

set $8 - 4x^3 = 0 \Rightarrow 4x^3 = 8$

$\Rightarrow x^3 = 2 \Rightarrow x = \sqrt[3]{2}$

$f(\sqrt[3]{2})$

$f(-1)$

$f(2)$

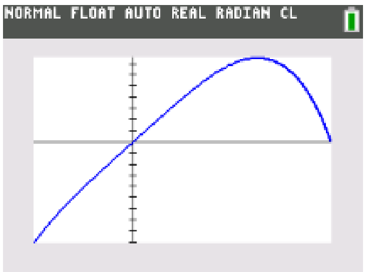
NORMAL FLOAT AUTO REAL RADIAN CL

X	Y1
1.2599	7.5595
-1	-9
2	0

X=

abs max at $x = \sqrt[3]{2}$

abs min at $x = -1$



NORMAL FLOAT AUTO REAL RADIAN CL

WINDOW
Xmin=-1
Xmax=2
Xscl=4
Ymin=-9
Ymax=7.55859375 → exact value
Yscl=1
Xres=1
ΔX=.01136363636363
TraceStep=.02272727272727

$f(\sqrt[3]{2}) = 8(\sqrt[3]{2}) - (\sqrt[3]{2})^4$

$= 2^3 \cdot 2^{\frac{1}{3}} - 2^{4/3}$

$= 2^{\frac{10}{3}} - 2^{4/3}$

$= 2^{4/3}(4 - 1)$

$= 3 \cdot \sqrt[3]{16}$

$= 3 \cdot 2\sqrt[3]{2}$

$= 6\sqrt[3]{2}$

NORMAL FLOAT AUTO REAL RADIAN CL

6³√(2)
7.559526299

$\sqrt[3]{2}\left(8 - \sqrt[3]{2^3}\right) =$

$\sqrt[3]{2}(8 - 2) = 6\sqrt[3]{2}$

Find the relative and absolute extrema for $y = x^{2/3}(x^2-4)$.

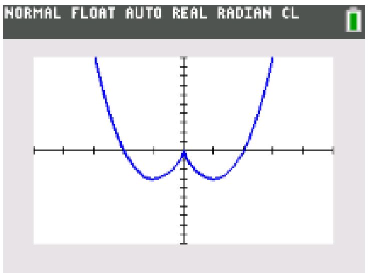
avoid product

$$= x^{8/3} - 4x^{2/3}$$

$$\frac{dy}{dx} = \frac{8}{3}x^{5/3} - \frac{8}{3}x^{-\frac{1}{3}}$$

$$= \frac{8}{3}x^{-1/3}\left(x^2-1\right) = \frac{8(x+1)(x-1)}{3\sqrt[3]{x}}$$

CV: $-1, 1, 0$



NORMAL FLOAT AUTO REAL RADIAN CL

X	Y_1
-1	-3
0	0
1	-3

X=

rel min at $(-1,-3)$ $(1,-3)$

rel max at $(0,0)$

abs min at $(-1,-3)$, $(1,-3)$

no abs extrema