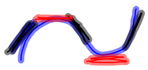
4.3 Increasing / Decreasing Functions



First Derivative Test:

Let c be a critical value for a function f(x) that is continuous on an open interval containing c. If f '(x) changes signs as x increases through c, then c is a relative extremum.

Example: Determine where f(x) is increasing, where f(x) is decreasing, then find its relative extrema.

$f(x) = 2x^3 + 3x^2 - 12x$

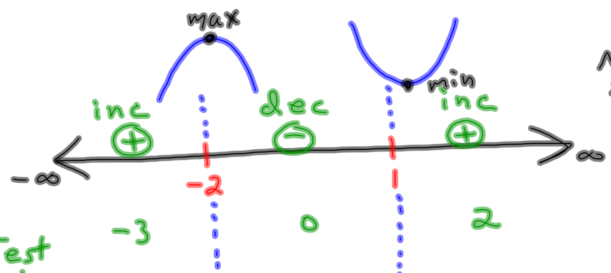
① find critical values

$f'(x) = 6x^2 + 6x - 12$

$= 6(x^2 + x - 2) = 6(x+2)(x-1).$

$f'(x) = 0$ when $x = -2, 1$ (CV's)

② find sign of $f'(x)$ as x increases through the CV's.



Number line showing domain of f(x)

Test Values (TV's)

Plug your TV's into $f'(x)$.

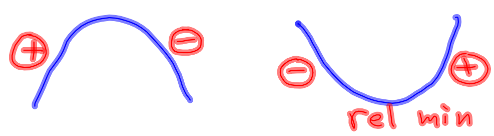
③ If $f'(TV) = \oplus / \ominus$, the TV is in an interval where f(x) is
increasing
decreasing

f(x) increasing when $x \in (-\infty, -2) \cup (1, \infty)$

decreasing when $x \in (-2, 1)$

rel max

④



rel max at $x = -2$

rel min at $x = 1$

rel max is $f(-2) = 20$ } plug into f(x) not f'(x)

$$f(x) = x + \frac{1}{x}$$
$$= x + x^{-1}$$

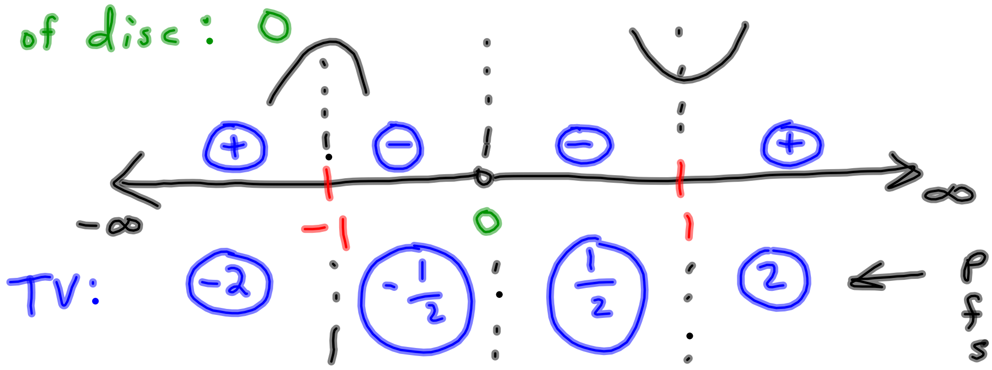
$$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$$

note 0 is not a cv. It's a point of discontinuity

If $1 - \frac{1}{x^2} = 0$ then $\frac{1}{x^2} = 1 \Rightarrow x^2 = 1$
$\Rightarrow x = \pm\sqrt{1} = \pm 1.$

cv: $-1, 1$

pt of disc: 0



plugged into f' to find slopes of tangent lines

inc: $x \in (-\infty, -1) \cup (1, \infty)$

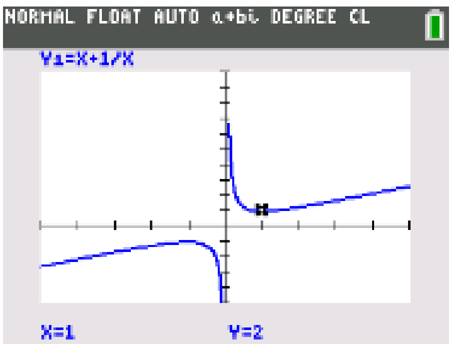
dec: $x \in (-1, 0) \cup (0, 1)$

Derivative tells you where (x-coord.).

To find y-coord. plug into $f(x)$ NOT $f'(x)$.

Rel max at $(-1, -2)$ $f(-1)$

Rel min at $(1, 2)$ $f(1)$



Jun 11-9:42 AM

$$f(x) = \frac{x^4+1}{x^2}$$

$$= \frac{x^4}{x^2} + \frac{1}{x^2} = \underline{x^2} + \underline{\underline{x^{-2}}}$$

$$f'(x) = 2x - 2x^{-3} = 2x - \frac{2}{x^3}$$

Note: 0 is a pt. of disc.

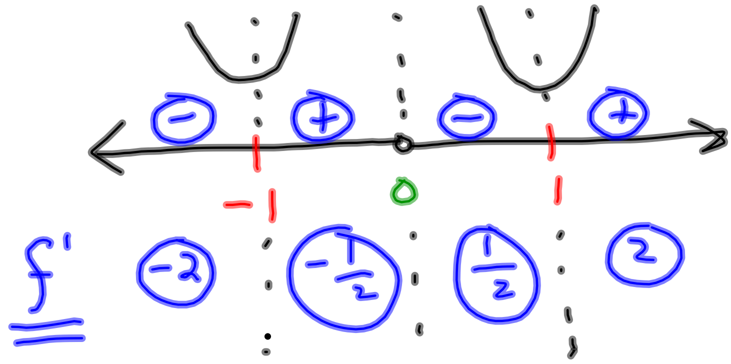
$$x^3\left(2x - \frac{2}{x^3}\right) = x^3(0) ;$$

$$2x^4 - 2 = 0 \Rightarrow 2x^4 = 2 \Rightarrow x^4 = 1.$$

$$x^4 - 1 = 0 \Rightarrow (x^2+1)(x^2-1) = 0 \Rightarrow (x^2+1)(x+1)(x-1) = 0$$

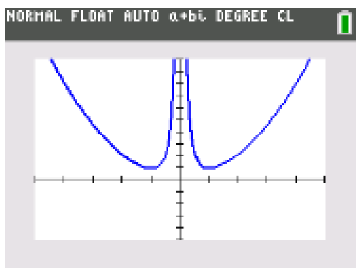
CV: $-1, 1$

pt of disc: 0



inc: $x \in (-1, 0) \cup (1, \infty)$

dec: $x \in (-\infty, -1) \cup (0, 1)$



No Rel Max

Rel Min @ $(-1, 2)$

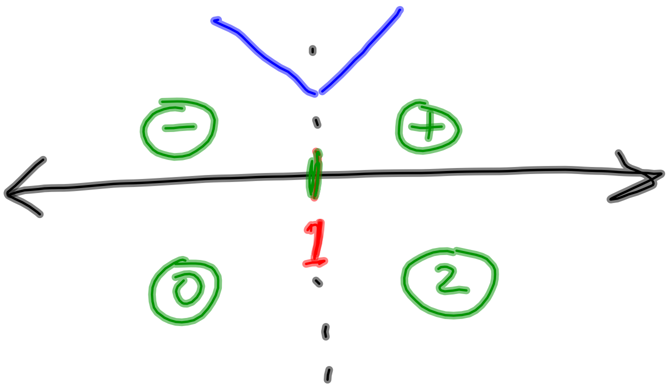
$(1, 2)$

$f(x) = (x-1)^{2/3}$

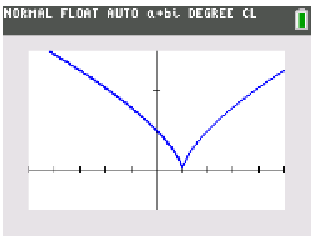
$f(1)$ exists
$f'(1)$ dne
$\therefore$ 1 is a cv

$f'(x) = \frac{2}{3}(x-1)^{-\frac{1}{3}}(1) = \dfrac{2}{3\sqrt[3]{x-1}}$

CV: 1



No Rel. Max
Rel Min @ (1, 0)

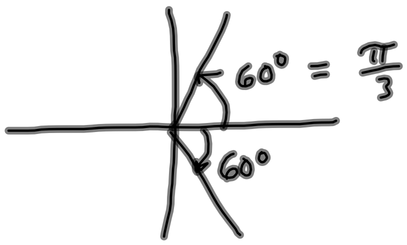


inc: $x \in (1, \infty)$
dec: $x \in (-\infty, 1)$

$f(x) = \frac{1}{2}x - \sin x$ on $\underline{(0, 2\pi)}$

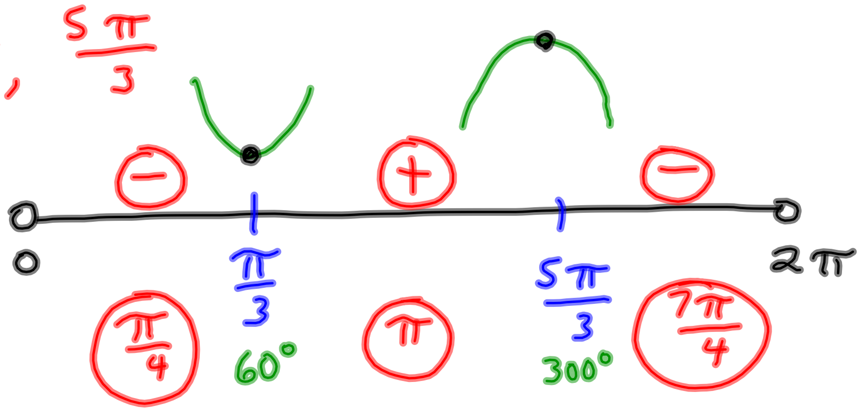
$f'(x) = \frac{1}{2} - \cos x$

$\cos x = \frac{1}{2}$



	$\pi/6$	$\pi/4$	$\pi/3$
θ	$30°$	$45°$	$60°$
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\sqrt{3}/2$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$

CV: $\frac{\pi}{3}, \frac{5\pi}{3}$



inc: $x \in \left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$

dec: $x \in \left(0, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right)$

Rel min at $\left(\frac{\pi}{3}, \frac{\pi}{6} - \frac{\sqrt{3}}{2}\right)$

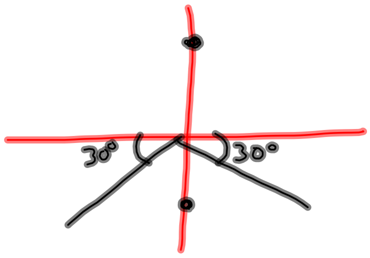
Rel max at $\left(\frac{5\pi}{3}, \frac{5\pi}{6} + \frac{\sqrt{3}}{2}\right)$

Jun 11-9:45 AM

$f(x) = \sin^2 x + \sin x$ on $(0, 2\pi)$

$f'(x) = 2\sin x\cos x + \cos x$

$= \cos x\,(2\sin x + 1)$



$\cos x = 0$

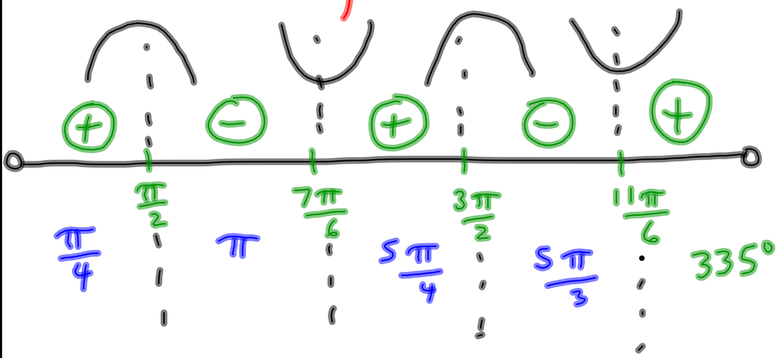
$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

	$\pi/6$ 30°	$\pi/4$ 45°	$\pi/3$ 60°
θ	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\sqrt{3}/2$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$



inc: $x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{7\pi}{6}, \frac{3\pi}{2}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right)$

dec: $x \in \left(\frac{\pi}{2}, \frac{7\pi}{6}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

$f(x) = \sin^2 x + \sin x$ on $(0, 2\pi)$

Rel max at $x = \frac{\pi}{2}, \frac{3\pi}{2}$ | $\left(\frac{\pi}{2}, 2\right), \left(\frac{3\pi}{2}, 0\right)$

Rel min at $x = \frac{7\pi}{6}, \frac{11\pi}{6}$ | $\left(\frac{7\pi}{6}, -\frac{1}{4}\right), \left(\frac{11\pi}{6}, -\frac{1}{4}\right)$

$\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)$