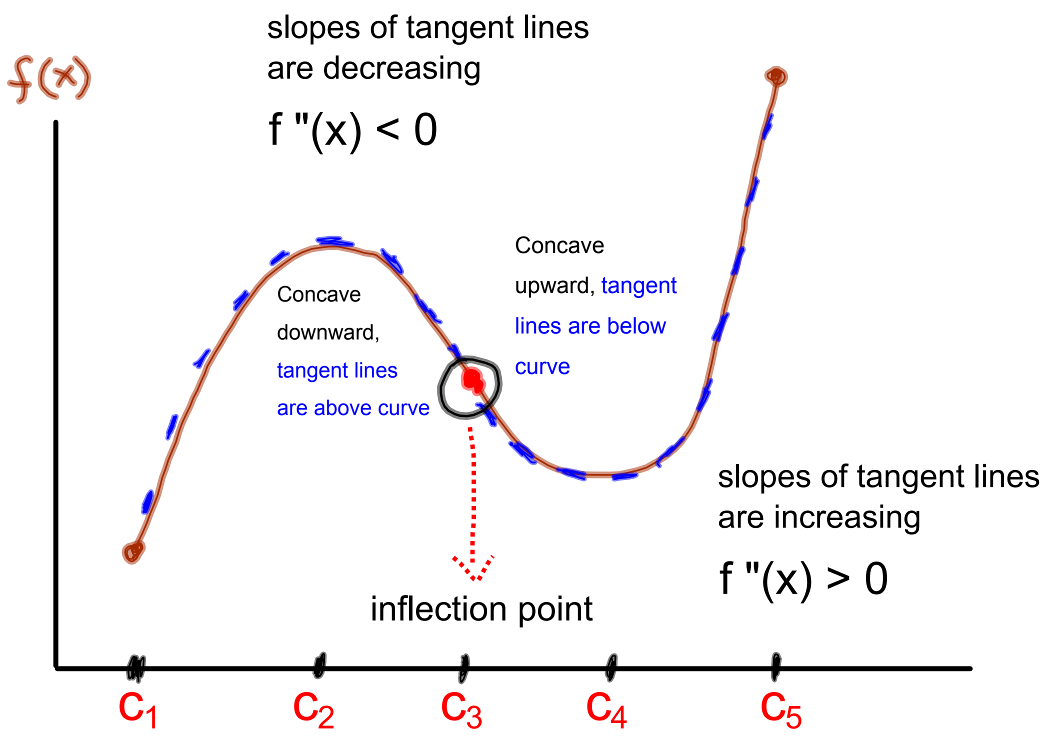
4.4 Concavity



If c is a point of inflection then f(c) exists and f "(c) =0. In addition,

If k is a relative extremum and

This is called the second derivative test

(1) f "(k) < 0, then there is a relative maximum at x = k.

(2) f "(k) > 0, then there is a relative minimum at x = k.

(3) f "(k) = 0, then one cannot say

Example: $f(x) = x^4 - 4x^3 = x^3(x-4)$

a) Find the x and y-intercepts $(0,0)$ + $(4,0)$

b) find local extrema $f'(x) = \underline{4x^3 - 12x^2} = 4x^2(x-3)$

$-$ $-$ $+$

-1 0 1 3 4

rel min at $(3,-27)$

C) find point of inflection(s) $f''(x) = 12x^2 - 24x$

$= 12x(x-2)$

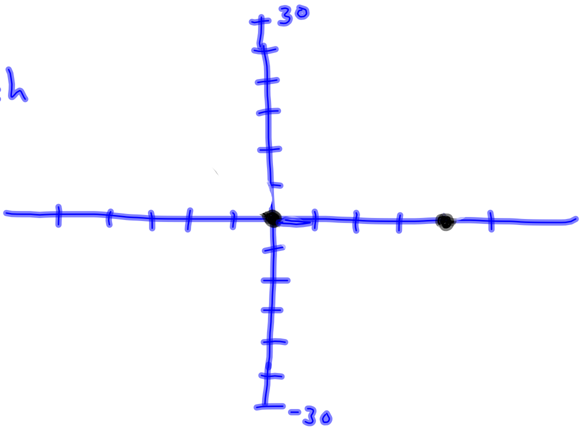
up down up

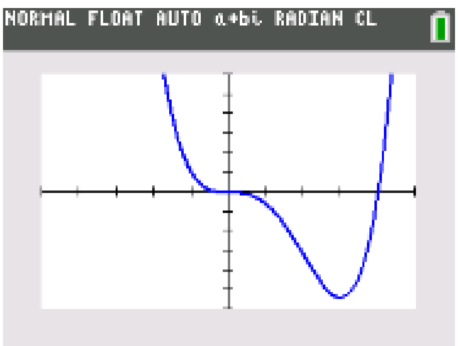
$+$ $-$ $+$

f'': -1 0 1 2 3

pts of infl.
$(0,0)$
$(2,-16)$

d) Sketch





Jun 18-1:32 PM

Consider $y = f(x) = \dfrac{6}{x^2+3} = 6(x^2+3)^{-1}$

1) find x and y-intercepts

No x-intercept

y-intercept: $(0, 2)$

2) find $f'(x) = -6(x^2+3)^{-2}(2x)$

$= -12x(x^2+3)^{-2} = \dfrac{-12x}{(x^2+3)^2}$

3) find relative extrema   cv: 0

$\oplus$ $\ominus$

-1 0 1

rel max $(0, 2)$

4) find $f''(x) = \dfrac{(x^2+3)^2(-12) - (-12x)\left[2(x^2+3)(2x)\right]}{(x^2+3)^4}$

$= \dfrac{-12(x^2+3)^2 + 48x^2(x^2+3)}{(x^2+3)^4} = \dfrac{-12(x^2+3)\left[(x^2+3) - 4x^2\right]}{(x^2+3)^4}$

$= \dfrac{-12(x^2+3)(3-3x^2)}{(x^2+3)^4} = \dfrac{-36(1-x^2)}{(x^2+3)^3} = \dfrac{-36(1+x)(1-x)}{(x^2+3)^3}$

5) find inflection points

cv'': $-1, 1$

UP DOWN UP

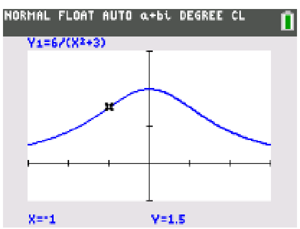
$\oplus$ $\ominus$ $\oplus$

-1 1

-2 0 2

pts of infl:

$\left(-1, \frac{3}{2}\right)$ & $\left(1, \frac{3}{2}\right)$



$f(x) = \frac{1}{x} = x^{-1}$

1) find x and y-intercepts

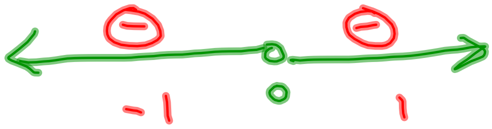
No x-int.

No y-int.

2) find $f'(x) = -x^{-2} = \frac{-1}{x^2} = -x^{-2}$

3) find relative extrema No CV

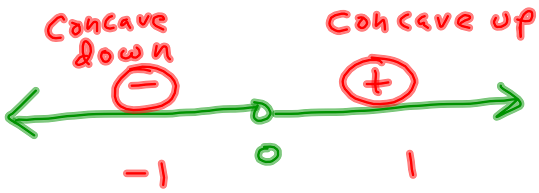
No rel. ext.



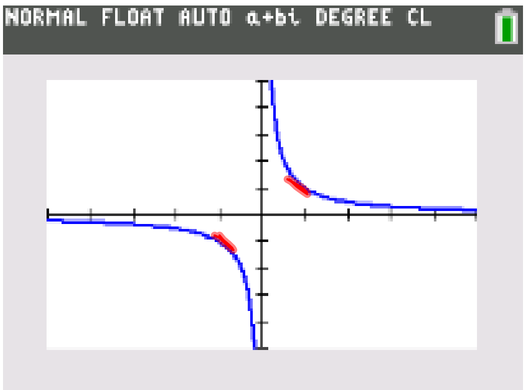
4) find $f''(x) = 2x^{-3} = \frac{2}{x^3}$

5) find inflection points No CV''

No infl. pts.



plug into f''



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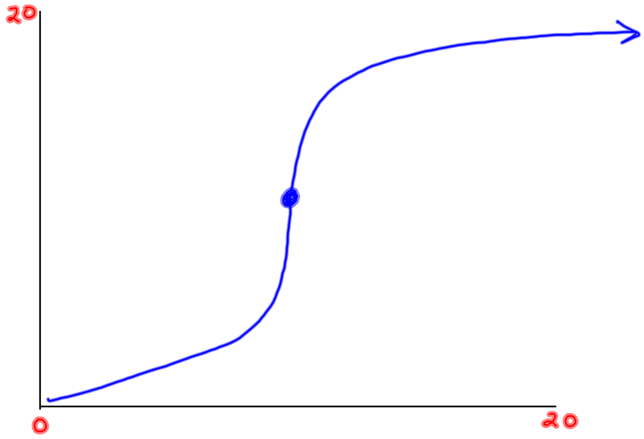
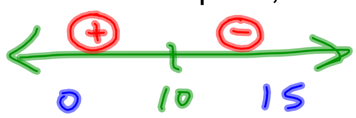
In business applications, the inflection point is called the point of diminishing returns.

Example: A company has determined that an investment of x hundred dollars in advertisement results in higher revenue. Say,

$R(x) = 4.5(x-10)^{1/3} + 10$ is the daily revenue in thousands of dollars.



Notice, its graph is an increasing function. Beyond the inflection point, the company gets less and less "bang" for buck.



$R'(x) = 1.5(x-10)^{-2/3}$

$R''(x) = -(x-10)^{-5/3} = \dfrac{-1}{\sqrt[3]{(x-10)^5}}$

pt of fl. is

(10, 10)

\$100 \$1000

\$1000/day ⇒

\$10,000 in rev./day

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