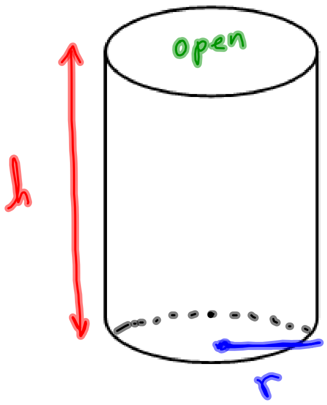
4.5 Applications

To save money and aluminum, we need to find the dimension of an open-top cylindrical container that has minimum surface area and holds 100 gallons (23,100 cubic inches)

$V = \pi r^2 h = 23100 \Rightarrow$



$h = \dfrac{23100}{\pi r^2}$

area of bottom

circumference

$S = \pi r^2 + 2\pi r h$

$S = \pi r^2 + 2\cancel{\pi} r \left(\dfrac{23100}{\cancel{\pi} r^2}\right)$

$S = \pi r^2 + \dfrac{46200}{r}$; $S = \pi r^2 + 46200 r^{-1}$

$\dfrac{dS}{dr} = 2\pi r - 46200 r^{-2}$

$= 2\pi r - \dfrac{46200}{r^2}$ | set to 0

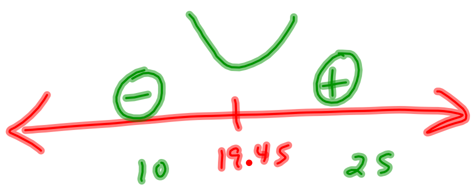
$2\pi r - \dfrac{46200}{r^2} = 0 \Rightarrow$

$2\pi r^3 - 46200 = 0 \Rightarrow$

$2\pi r^3 = 46200 \Rightarrow$

$r^3 = \dfrac{46200}{2\pi} \Rightarrow r = \sqrt[3]{\dfrac{23100}{\pi}} \approx 19.446 \text{ in}$

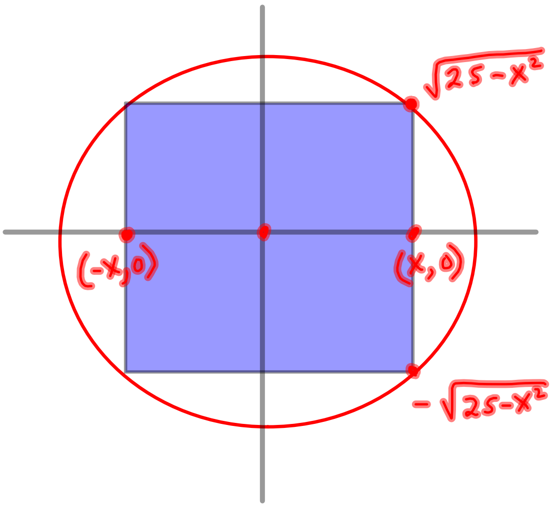
$h = \dfrac{23100}{\pi r^2} = \dfrac{23100}{\pi (19.446)^2} \approx 19.446$



It's a Min

thus, the radius & the height are the same.

Find the largest rectangle that can fit inside the circle described by the equation $x^2 + y^2 = 25$ $\Rightarrow$ $y^2 = 25 - x^2$; $y = \pm\sqrt{25-x^2}$



$A = lw$

$A = (2x)\left(2\sqrt{25-x^2}\right)$

$A = \underline{4x}\,\underline{(25-x^2)^{\frac{1}{2}}}$

Domain: $x \in (0,5)$

$$A' = 4x\left[\tfrac{1}{2}(25-x^2)^{-\frac{1}{2}}(-2x)\right] + (25-x^2)^{\frac{1}{2}}(4)$$

$$= -4x^2(25-x^2)^{-\frac{1}{2}} + 4(25-x^2)^{\frac{1}{2}} = 0$$

Set A' to 0: $-4x^2(25-x^2)^{-\frac{1}{2}} = -4(25-x^2)^{\frac{1}{2}}$

$$\sqrt{25-x^2}\,\frac{x^2}{\sqrt{25-x^2}} = \sqrt{25-x^2}\cdot\sqrt{25-x^2}$$

$$x^2 = 25 - x^2$$

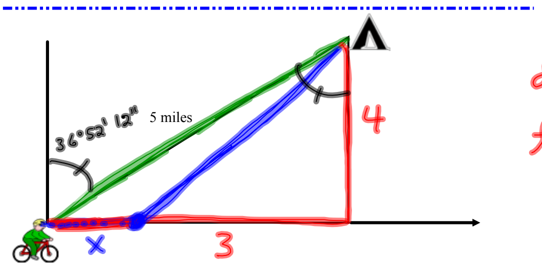
$$2x^2 = 25 \Rightarrow x^2 = \frac{25}{2}$$

$$\Rightarrow x = \frac{5}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

$l = 2x = 5\sqrt{2}$

$w = 2\sqrt{25-x^2} = 2\sqrt{25-\frac{25}{2}} = 2\sqrt{\frac{25}{2}} = \frac{2\cdot 5}{\sqrt{2}} = 5\sqrt{2}$

A bicyclist pedals due East on a paved road that is South of and parallel to a river. His campsite near the river is 5 miles away and 36°52'12' East of North of his position. If he stays on the paved road, his rate is 30 mph. If he leaves the paved road, his rate is 10 mph. What is the minimum number of minutes it would take for him to reach his campsite?



$d = rt$

$t = \frac{d}{r}$

① $t = \frac{5\text{mi}}{10\text{mph}} = \frac{1}{2}\text{hr} = 30$ minutes

② $t = \frac{3\text{mi}}{30\text{mph}} + \frac{4\text{mi}}{10\text{mph}} = .1\text{hr} + .4\text{hr} = .5\text{hr} = 30$ min

③ $t = \frac{x}{30} + \frac{\text{hypotenuse}}{10}$

hypotenuse $= \sqrt{16 + (3-x)^2}$

$t(x) = \frac{1}{30}x + \left(\underbrace{16+9}_{25} - 6x + x^2\right)^{\frac{1}{2}}/10$

$t' = \frac{1}{30} + \frac{1}{2}(x^2 - 6x + 25)^{-\frac{1}{2}}(2x-6)/10$ $\quad 2(x-3)$

$t' = \frac{1}{30} + \frac{(x-3)(x^2-6x+25)^{-\frac{1}{2}}}{10} = 0$

$\frac{1}{30} = \frac{-(x-3)}{10\sqrt{x^2-6x+25}} \Rightarrow 10\sqrt{x^2-6x+25} = -30(x-3)$

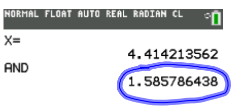
$\left(\sqrt{x^2-6x+25}\right)^2 = \left(-3(x-3)\right)^2 \Rightarrow x^2 - 6x + 25 = 9(x-3)^2$

$x^2 - 6x + 25 = 9(x^2 - 6x + 9)$

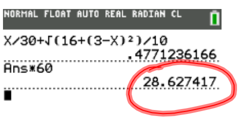
$x^2 - 6x + 25 = 9x^2 - 54x + 81$

$-x^2 + 6x - 25 \quad -x^2 + 6x - 25$

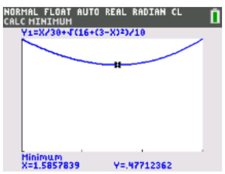
$0 = 8x^2 - 48x + 56 \Rightarrow x^2 - 6x + 7 = 0$



$t(1.586)$



28 min and 38 sec



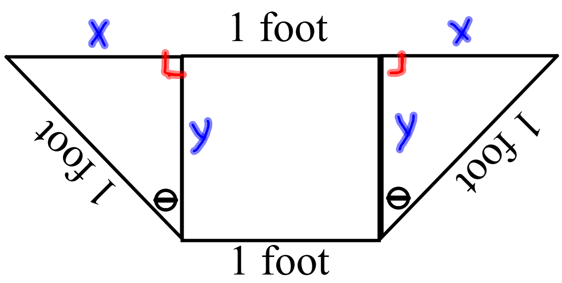
Jun 17-8:54 AM

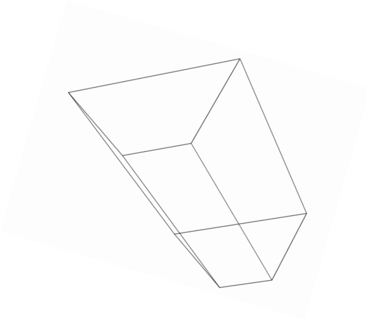
The design for constructing a trough are so that the cross section must have the following specifications. Find the value of theta so that the volume of the trough is maximum.

$\sin\theta = \frac{x}{1}$

$\cos\theta = \frac{y}{1}$

$x = \sin\theta$
$y = \cos\theta$





$$A = 2\left[\tfrac{1}{2}xy\right] + y(1) \;; \quad A = xy + y$$

$$A = \sin\theta\cos\theta + \cos\theta$$

$$A' = \sin\theta(-\sin\theta) + \cos\theta(\cos\theta) - \sin\theta$$

$$= \cos^2\theta - \sin^2\theta - \sin\theta$$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$$\cos^2\theta - \sin^2\theta - \sin\theta = 0$$

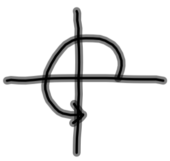
$$(1 - \sin^2\theta) - \sin^2\theta - \sin\theta = 0$$

$$-2\sin^2\theta - \sin\theta + 1 = 0 \;\Rightarrow\; 2\sin^2\theta + \sin\theta - 1 = 0$$

$$(2\sin\theta - 1)(\sin\theta + 1) = 0$$

$2\sin\theta - 1 = 0$
$2\sin\theta = 1$
$\sin\theta = \frac{1}{2}$

$\sin\theta + 1 = 0$
$\sin\theta = -1$
$\theta = \cancel{270^\circ = \frac{3\pi}{2}}$



ANSWER: $\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$ or $\frac{\pi}{6}$

Jun 17-12:29 PM

The daily demand function for producing n widgets is $p(n) = \frac{50}{\sqrt{n}}$

dollars. The cost for making n widget is linear. The daily fixed cost is $500 and the cost of making each additional widget (variable cost) is 50 cents. What price would generate maximum profit.

$$P(n) = R(n) - C(n) \quad ; \quad R(n) = n\,p(n)$$

$$= n \cdot \frac{50}{\sqrt{n}}$$

$$= 50\sqrt{n}$$

$$C(n) = mn + b$$

$$= \frac{1}{2}n + 500$$

$$P(n) = 50n^{\frac{1}{2}} - \left(\frac{1}{2}n + 500\right)$$

$$= 50n^{\frac{1}{2}} - \frac{1}{2}n - 500$$

marginal profit

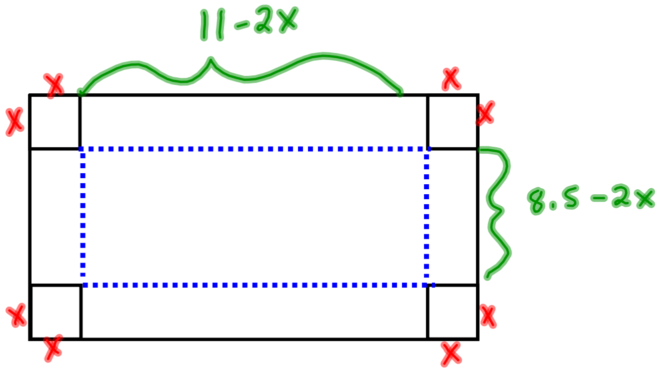
$$P'(n) = 25n^{-\frac{1}{2}} - \frac{1}{2}$$

$$25n^{-\frac{1}{2}} - \frac{1}{2} = 0 \Rightarrow \frac{25}{\sqrt{n}} = \frac{1}{2}$$

$$\Rightarrow \sqrt{n} = 50 \Rightarrow n = 2500$$

$$\text{price} = \frac{50}{\sqrt{2500}} = \frac{50}{50} = \$1.00$$

An open box is created by cutting out squares from the corners of a piece of paper that is 8.5 inches by 11 inches then folding up the sides. What is the maximum volume for such a box.



$V = lwh$

$V = (11-2x)(8.5-2x)\,x$

$= x(93.5 - 22x - 17x + 4x^2)$

$= x(93.5 - 39x + 4x^2)$

$= 4x^3 - 39x^2 + 93.5x$

$V' = 12x^2 - 78x + 93.5 = 0$

NORMAL FLOAT AUTO REAL RADIAN CL

X= ~~4.91458203~~

AND 1.58541797

let $x \approx 1.59$ in

$V'' = -24x - 78$

Jun 18-8:52 AM