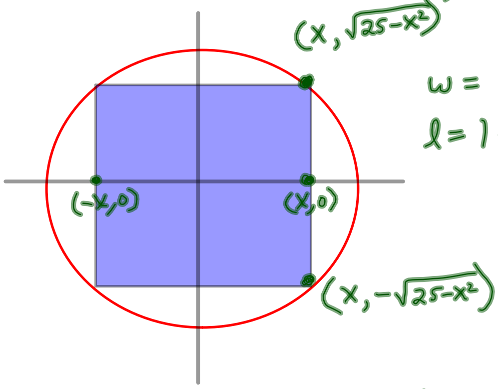
4.5 Applications

Ex.1: Do not cover in class for time purposes

Find the largest rectangle that can fit inside the circle described by the equation $x^2 + y^2 = 25$ $\Rightarrow y^2 = 25 - x^2 \Rightarrow y = \pm\sqrt{25-x^2}$



$w =$ width $= x - (-x) = 2x$

$l =$ length $= \sqrt{25-x^2} - (-\sqrt{25-x^2})$

$= 2\sqrt{25-x^2}$

$A = lw = (2x)(2\sqrt{25-x^2})$

$A(x) = \underbrace{4x}_{1^{st}} \cdot \underbrace{(25-x^2)^{\frac{1}{2}}}_{2^{nd}}$

$A'(x) = 4x\left[\frac{1}{2}(25-x^2)^{-\frac{1}{2}}(-2x)\right] + (25-x^2)^{\frac{1}{2}}(4)$

$= -4x^2(25-x^2)^{-\frac{1}{2}} + 4(25-x^2)^{\frac{1}{2}}$

$= -4(25-x^2)^{-\frac{1}{2}}\left[x^2 - (25-x^2)\right]$

$= -4(25-x^2)^{-\frac{1}{2}}(2x^2-25)$

$= \dfrac{-4(2x^2-25)}{\sqrt{25-x^2}}$ $\quad 0 < x < 5$

Consider $2x^2 - 25 = 0 \Rightarrow x^2 = \frac{25}{2} \Rightarrow$

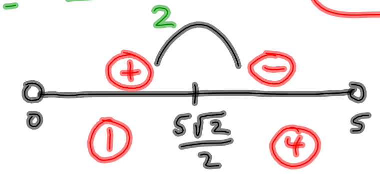
$x = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

Width $= 2x = 5\sqrt{2}$

Length $= 2\sqrt{25-x^2} = 2\sqrt{25 - \frac{25}{2}}$

$= 2\sqrt{\frac{50}{2} - \frac{25}{2}} = 2\sqrt{\frac{25}{2}} = 2 \cdot \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{(2)(5)(\sqrt{2})}{2} = 5\sqrt{2}$

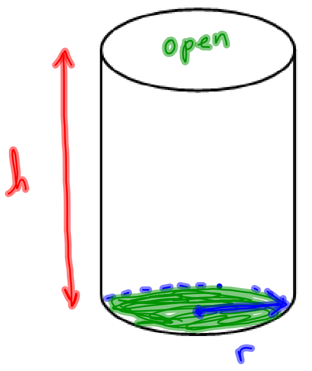


Max area $=$

$lw = (5\sqrt{2})(5\sqrt{2})$

$= 50$

To save money and aluminum, we need to find the dimension of an open-top cylindrical container that has minimum surface area and holds 100 gallons (23,100 cubic inches)



$$S = \pi r^2 + 2\pi r h$$

$$V = \pi r^2 h = 23100$$

$$h = \frac{23100}{\pi r^2}$$

$$S = \pi r^2 + 2\cancel{\pi} r \left(\frac{23100}{\cancel{\pi} r^2}\right)$$

$$S = \pi r^2 + \frac{46200}{r} \quad ; \quad S = \pi r^2 + 46200 r^{-1}$$

$$\frac{dS}{dr} = 2\pi r - 46200 r^{-2}$$

$$= 2\pi r - \frac{46200}{r^2}$$

Set to 0: $r^2\left(2\pi r - \frac{46200}{r^2}\right) = 0(r^2)$

$$2\pi r^3 - 46200 = 0$$

$$2\pi r^3 = 46200 \Rightarrow$$

$$r^3 = \frac{46200}{2\pi} \Rightarrow r = \sqrt[3]{\frac{23100}{\pi}}$$

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NORMAL FLOAT AUTO REAL RADIAN CL
³√(23100/π)
                    19.44557109
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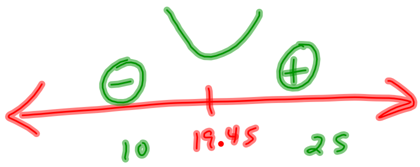
$= r$



$$h = \frac{23100}{\pi r^2} = 19.445$$

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NORMAL FLOAT AUTO REAL RADIAN CL
³√(23100/π)
                    19.44557109
23100/(π*Ans²)
                    19.44557109
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thus, the radius & the height are the same.