4.7 Antiderivatives

Definition: F(x) is an antiderivative of f(x) if F '(x) = f(x).

Example: Consider f(x) = $3x^2 + 2x - 1$

Antiderivatives for f(x) include

$$F(x) = x^3 + x^2 - x + 7$$
$$G(x) = x^3 + x^2 - x - 9$$
$$H(x) = x^3 + x^2 - x + 100$$

The generalized antiderivative for f(x) is called the indefinite integral.

The indefinite integral for this particular function is

$$\int f(x)\,dx = x^3 + x^2 - x + C.$$

C is any constant. The process of finding anti derivatives is called integrating.

Most Common Rule: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

if $n \neq -1$

Ex. find $\int (x^3 + 4x^2 - x + \overset{3x^0}{3})\,dx$

$\frac{x^4}{4} + \frac{4x^3}{3} - \frac{x^2}{2} + 3x + C$

find $\int \frac{4}{x^5} \cdot dx = 4\int x^{-5}\,dx = \frac{4 \cdot x^{-4}}{-4} + C$

$= -1 \cdot x^{-4} + C = \frac{-1}{x^4} + C$

find $\int \frac{dx}{\sqrt{x}} = \int x^{-\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= 2\sqrt{x} + C$

find $\int \overset{3x^0}{3}\,dx = 3x + C$

find $\int dx = \int 1 \cdot dx = \int x^0 \cdot dx = x + C$

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More Rules:

$$\int \sin kx \, dx = \frac{-\cos kx}{k} + C$$

$$\int \cos kx \, dx = \frac{\sin kx}{k} + C$$

$$\int \sin 7x \, dx = \frac{-\cos 7x}{7} + C$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\int \frac{\sin x}{\cos^2 x} dx = \int \frac{\sin x \cdot 1}{\cos x \cdot \cos x} dx$$

$$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx = \int \tan x \sec x \, dx$$

$$= \sec x + C$$

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$$\int \frac{2-\sin^2 x}{\cos^2 x}\,dx =$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\int \frac{1+(1-\sin^2 x)}{\cos^2 x}\,dx = \int \frac{1+\cos^2 x}{\cos^2 x}\,dx =$$

$$\int \left(\frac{1}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x}\right) dx = \int (\sec^2 x + 1)\,dx = \tan x + x + C$$

$$\int w^2\left(1-\sqrt{w}\right) dw = \int \left(w^2 - w^{5/2}\right) dw =$$

$$\sqrt{w} \to w^{\frac{1}{2}}$$

$$\frac{w^3}{3} - \frac{w^{7/2}}{7/2} + C = \frac{1}{3}w^3 - \frac{2}{7}w^{7/2} + C$$

Ex. On the moon, the acceleration due to gravity is -5.3 feet per sec^2. An astronaut shoots a bullet straight up. After 2 minutes its velocity $= \frac{dy}{dt}$ is 64 feet per second and its height is 8.683 miles. = 45,846.24 feet

a) What is the initial velocity of the bullet?

b) How long does it take for the bullet to hit the ground?

$s(t)$ = displacement = height

$v(t) = s'(t)$

$a(t) = v'(t) = s''(t)$

$v(t) = \int a(t)\,dt$

$s(t) = \int v(t)\,dt$

a) $a(t) = -5.3 \Rightarrow v(t) = \int -5.3\,dt = -5.3t + C$

$v(120) = -5.3(120) + C$
$v(120) = 64$

$-5.3(120) + C = 64$
$-636 + C = 64 \Rightarrow C = 700$

$v(t) = -5.3t + 700$; $v(0) =$ 700 ft/sec

b) $s(t) = \int v(t)\,dt = \int(-5.3t + 700)\,dt$

$= \frac{-5.3t^2}{2} + 700t + k$

$s(120) = \frac{-5.3(120)^2}{2} + 700(120) + k$
$s(120) = 45{,}846.24$ feet

$45840 + k = 45{,}846.24$
$k = 6.24$ ft

$s(t) = \frac{-5.3t^2}{2} + 700t + 6.24 = 0$

NORMAL FLOAT AUTO REAL RADIAN CL
X= -.0089139849
AND 264.1598574

$t \approx 264$ sec

4 min, 24 sec

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