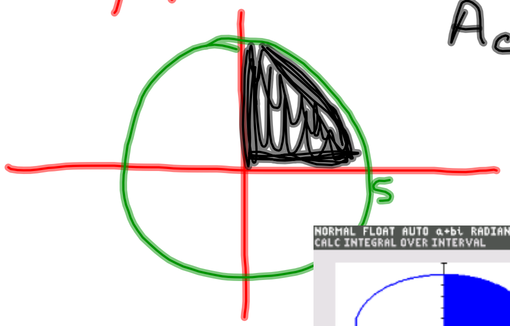
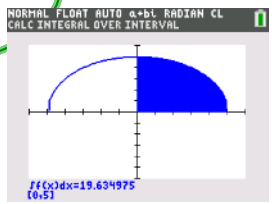
$$\int_0^5 \sqrt{25-x^2}\,dx$$

$x^2+y^2=25$

Center $(0,0)$, $r=5$

$y=\pm\sqrt{r^2-x^2}$



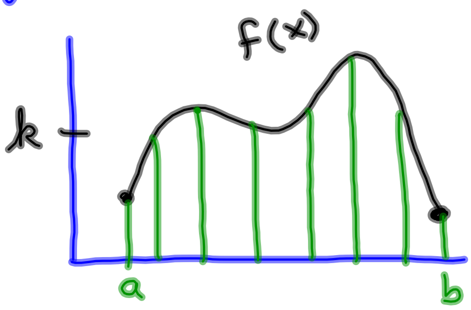


$A_c = \pi r^2$

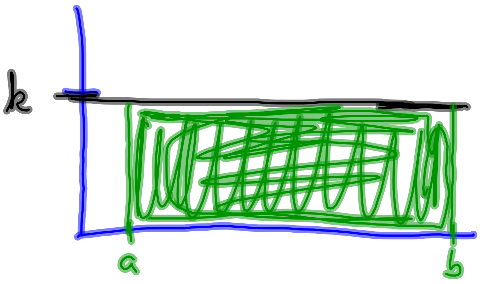
$$A = \frac{1}{4}\pi(25)$$

$$= \frac{25\pi}{4} \approx 19.63$$

Average value of a function on $[a,b]$



If I Knew the average length of each green line, say $g(x)=k$



then $\int_a^b f(x)\,dx =$

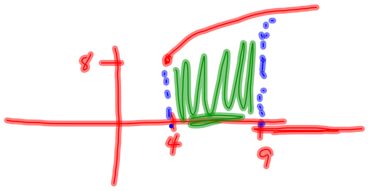
$k(b-a)$

k = avg length of green lines = avg value of $f(x)$ on $[a,b]$

Thus, $$k = \frac{\int_a^b f(x)\,dx}{b-a} \Rightarrow$$

$$\text{average of } f(x) \text{ on } [a,b] = \frac{\int_a^b f(x)\,dx}{b-a}$$

Ex. What is the average value of $f(x)$ on $[4, 9]$ if $f(x) = 4\sqrt{x}$



$$Avg = \frac{\int_4^9 4\sqrt{x}\,dx}{9-4}$$

$$= \frac{4\int_4^9 x^{1/2}\,dx}{5} = \frac{\frac{4x^{3/2}}{3/2}}{5}\Bigg|_4^9 = \frac{8}{15}x^{3/2}\Bigg|_4^9$$

$$= \frac{8}{15}\left[27 - 8\right] = \frac{8}{15}(19) = \frac{152}{15} = 10\tfrac{2}{15}$$

NORMAL FLOAT AUTO REAL RADIAN CL

fnInt(4√(X),X,4,9)
50.66666667
Ans/5
10.13333333
Ans▸Frac
152/15

Ex. What is the average value of $f(x)$ on $\left[\frac{-\pi}{2}, \frac{\pi}{4}\right]$ if $f(x) = \cos x$

$-1.57, .79$

$$avg = \frac{\int_{-\pi/2}^{\pi/4} \cos x\,dx}{\frac{\pi}{4} - \left(\frac{-\pi}{2}\right)} = \frac{\sin x \Big|_{-\pi/2}^{\pi/4}}{\frac{\pi}{4} + \frac{2\pi}{4}}$$

$$= \frac{\sin\frac{\pi}{4} - \sin\left(\frac{-\pi}{2}\right)}{\frac{3\pi}{4}} = \frac{4\left(\sqrt{2}/2 - (-1)\right)}{4\left(3\pi/4\right)}$$

$$= \frac{2\sqrt{2}+4}{3\pi} = \frac{2(\sqrt{2}+1)}{3\pi} \approx 0.7245$$

random numbers
$-1.25, -1, .5, 0, .6, -.5$

NORMAL FLOAT AUTO REAL RADIAN CL

L1	L2	L3	L4	L5	2
-1.25	.31532	------	------	------	
-1	.5403				
.5	.87758				
0	1				
.6	.82534				
-.5	.87758				
------	------				

L2(1)=.31532236239527

NORMAL FLOAT AUTO REAL RADIAN CL

1-Var Stats
x̄=.7393542345
Σx=4.436125407
Σx²=3.61283596
Sx=.2580573009
σx=.235573008
n=6
minX=.31532236
↓Q1=.5403023059

$\bar{x} \approx .739$

Ex. Find a cubic function so that

* $f'(3) = 0$, $f'(-4) = 0$,
* $f''(3) < 0$, $f''(-4) > 0$, and
* $f(0) = 11$.

* $f'(x) = k(x-3)(x+4) = k(x^2+x-12)$

* $f''(x) = k(2x+1)$ since $f''(3) < 0$, try $k=-1$

$= -(2x+1)$

* $f'(x) = -(x^2+x-12) = -x^2-x+12$

$f(x) = \int f'(x)\,dx = \int (-x^2-x+12)\,dx$

$= \frac{-x^3}{3} - \frac{x^2}{2} + 12x + C$

$f(0) = 11$

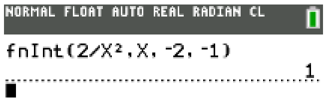
$f(0) = \frac{-0^3}{3} - \frac{0^2}{2} + 12(0) + C = C \Rightarrow C = 11$

$f(x) = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 12x + 11$

Jun 23-9:19 AM

Find $\int_{-2}^{-1} \frac{2}{x^2}\,dx = 2\int_{-2}^{-1} x^{-2}\,dx =$

$$\left.\frac{2x^{-1}}{-1}\right|_{-2}^{-1} = 2\left[\frac{-1}{x}\right]_{-2}^{-1} = 2\left[1-\frac{1}{2}\right] = 2\left(\frac{1}{2}\right) = 1$$



Find $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \csc^2 x\,dx = -\cot x \Big|_{\pi/6}^{5\pi/6}$

$\sin 150° = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$

$\cos 150° = -\sqrt{3}/2$

$\cos 30° = \sqrt{3}/2$

$$-\left[\cot\frac{5\pi}{6} - \cot\frac{\pi}{6}\right] =$$

$$-\left[-\sqrt{3} - \sqrt{3}\right] = -\left[-2\sqrt{3}\right] = 2\sqrt{3}$$

$\cos(-x) = \cos(x)$

even

Find $\int_{-\frac{\pi}{3}}^{\pi/3} \frac{1-\cos 2t}{2}\,dt = 2\int_0^{\pi/3}\left(\frac{1}{2} - \frac{\cos 2t}{2}\right)dt$

$$\int_0^{\frac{\pi}{3}} (1-\cos 2t)\,dt = \left.\left(t - \frac{\sin 2t}{2}\right)\right|_0^{\pi/3} =$$

$$\left(\frac{\pi}{3} - \frac{\sin 2\pi/3}{2}\right) - (0-0) =$$

$$\frac{\pi}{3} - \frac{\sqrt{3}/2}{2} = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

Jun 23-9:20 AM

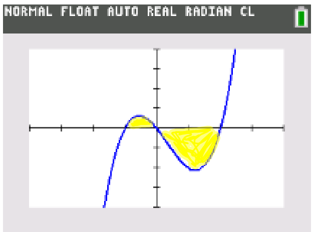
Find $\int_4^9 \frac{1-\sqrt{u}}{\sqrt{u}}\,du = \int_4^9 \left(\frac{1}{\sqrt{u}} - \frac{\sqrt{u}}{\sqrt{u}}\right) du =$

$$\int_4^9 \left(u^{-\frac{1}{2}} - 1\right) du = \left.\frac{u^{\frac{1}{2}}}{\frac{1}{2}} - u \right|_4^9 =$$

$$\left. 2\sqrt{u} - u \right|_4^9 = (6-9) - (4-4)$$

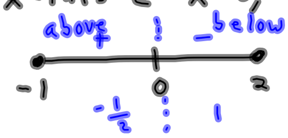
$$= -3$$

Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x$ on $[-1, 2]$.

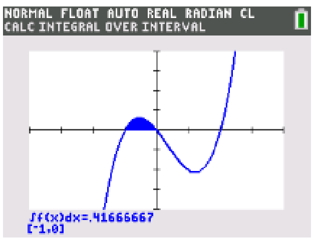


$$= x(x^2 - x - 2) = x(x-2)(x+1)$$

x-ints @ $x = 0, 2, -1$

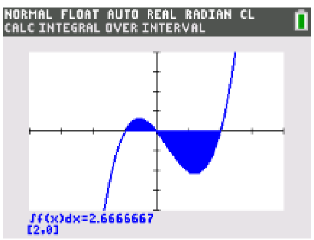


$$A = \int_{-1}^{0} f(x)\,dx + \int_{2}^{0} f(x)\,dx$$



NORMAL FLOAT AUTO REAL RADIAN CL

.416666666666666▸Frac
5/12



NORMAL FLOAT AUTO REAL RADIAN CL

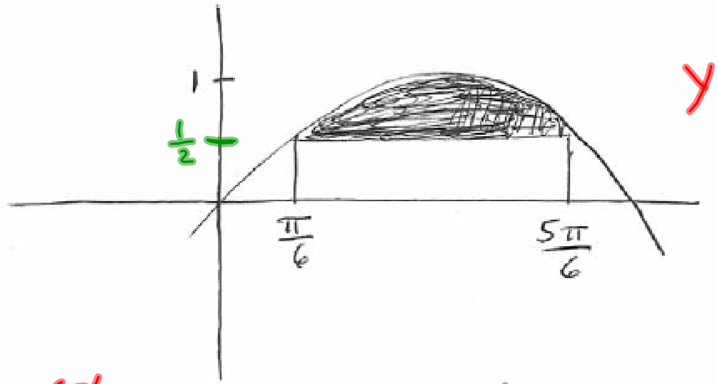
2.66666666666▸Frac
8/3

$$A = \frac{5}{12} + \frac{8}{3}$$

$$= \frac{5}{12} + \frac{32}{12}$$

$$= \frac{37}{12} = 3\frac{1}{12}$$

Find the area of the shaded region



$y = \sin x$

$A_{Rect} = lw =$

$\frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) =$

$\frac{1}{2}\left(\frac{2\pi}{3}\right) = \frac{\pi}{3}$

$$\int_{\pi/6}^{5\pi/6} \sin x \, dx = -\cos x \Big|_{\frac{\pi}{6}}^{5\pi/6}$$

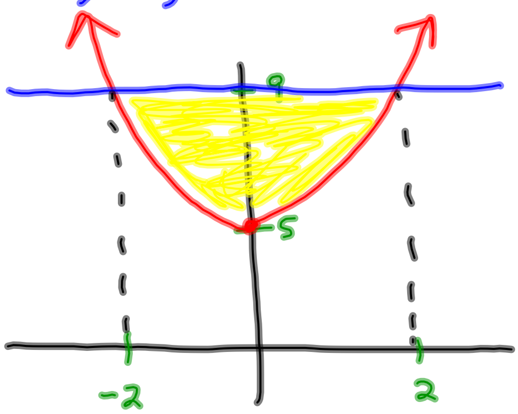
$$= -\left[\cos\frac{5\pi}{6} - \cos\frac{\pi}{6}\right] = -\left[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right]$$

$$= \sqrt{3}$$

$$A_{shaded} = \sqrt{3} - \frac{\pi}{3}$$

Ex. Let $f(x) = x^2 + 5$. Find the area between $f(x)$ and

a) $y = 9$



intersection of x^2+5, 9

$x^2 + 5 = 9 \Rightarrow$

$x^2 = 4 \Rightarrow x = \pm 2$

$$2\int_0^2 (x^2+5)\,dx = 2\left[\frac{x^3}{3} + 5x\right]_0^2$$

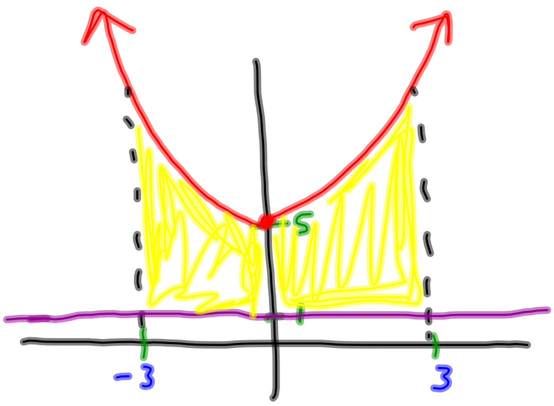
$$= 2\left[\left(\frac{8}{3} + 10\right) - (0+0)\right]$$

$$= 2\left(\frac{38}{3}\right) = \frac{76}{3}$$

$A_{yellow} = 36 - \frac{76}{3}$

$= \frac{108}{3} - \frac{76}{3} = \frac{32}{3} = 10\frac{2}{3}$

b) $y = 1$, $x = -3$, $x = 3$



$A_{Rect} = 6$

$2\int_0^3 (x^2+5)\,dx =$

$2\left[\frac{x^3}{3} + 5x\right]_0^3 =$

$2\left[(9+15) - (0+0)\right] =$

$2(24) = 48$

$A_{yellow} = 48 - 6 = 42$