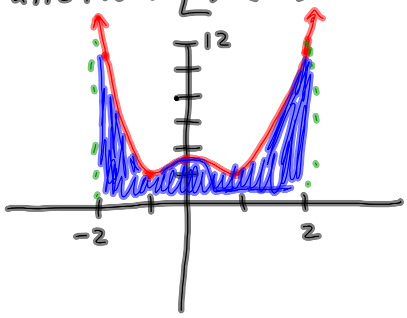
5.3 and 5.4 Definite Integrals

Fundamental Theorem of Calculus:

If $f(x) \geq 0$ on $[a, b]$, then the area between the x-axis and the curve $f(x)$ where $x \in [a, b]$ is

$$\int_a^b f(x)\,dx = F(b) - F(a), \quad F'(x) = f(x).$$

Example for even function $[f(-x) = f(x)]$



$$A = \int_{-2}^{2} (x^4 - 2x^2 + 3)\,dx$$

$$\frac{x^5}{5} - \frac{2x^3}{3} + 3x \Big|_{-2}^{2}$$

$$\left(\frac{32}{5} - \frac{16}{3} + 6\right) - \left(\frac{-32}{5} + \frac{16}{3} - 6\right) = 14\tfrac{2}{15}$$

$$A = 2\int_0^2 (x^4 - 2x^2 + 3)\,dx = 2\left(\frac{32}{5} - \frac{16}{3} + 6\right) - 0$$

$$= 14\tfrac{2}{15}$$

note: $\int_2^{-2} f(x)\,dx = -14\tfrac{2}{15}$

note: $A = \int_{-2}^{0} f(x)\,dx + \int_0^2 f(x)\,dx$

In general, $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$ where $b \in [a, c]$.

reversed

Also, $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

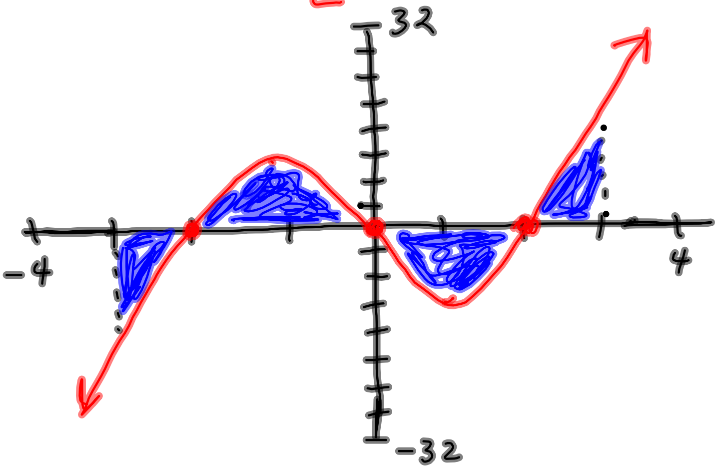
> If $f(x)$ is an even function,
> $$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

Jun 23-9:18 AM

Example of an odd function.

Ex. Let $f(x) = 2x^3 - 8x$ $[f(-x) = -f(x)]$

Find $\int_{-3}^{3} f(x)\,dx$



$$\frac{2x^4}{4} - \frac{8x^2}{2}\Big|_{-3}^{3} =$$

$$\frac{x^4}{2} - 4x^2\Big|_{-3}^{3} = \left(\frac{81}{2} - 36\right) - \left(\frac{81}{2} - 36\right) = 0$$

If $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)\,dx = 0$

The area in blue is $\left|\int_{-3}^{-2} f(x)\,dx\right| + \left|\int_{-2}^{0} f(x)\,dx\right| + \left|\int_{0}^{2} f(x)\,dx\right| + \left|\int_{2}^{3} f(x)\,dx\right|$

$$\int_{-2}^{0} f(x)\,dx = \frac{x^4}{2} - 4x^2\Big|_{-2}^{0} = 0 - (8 - 16) = \underline{\underline{8}}$$

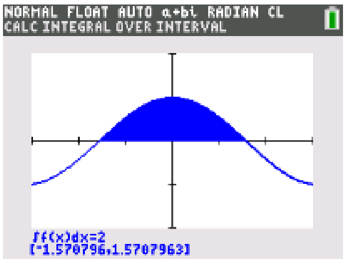
$$\int_{2}^{3} f(x)\,dx = \frac{x^4}{2} - 4x^2\Big|_{2}^{3} = \left(\frac{81}{2} - 36\right) - (8 - 16)$$

$$= 4.5 + 8 = \underline{\underline{12.5}}$$

$$A = 2(8) + 2(12.5) = 41$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx = \sin x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2 \sin x \Big|_{0}^{\pi/2}$$

even function

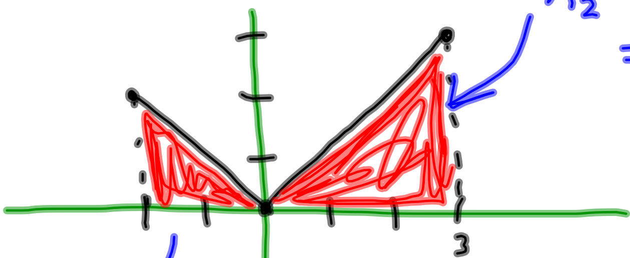


$$= \sin\frac{\pi}{2} - \left[\sin -\frac{\pi}{2}\right]$$

$$= 1 - (-1) = 2$$

Sometimes it's best to use geometry over antiderivatives.

Find $\displaystyle\int_{-2}^{3} |x| \, dx = \int_{-2}^{0} -x \, dx + \int_{0}^{3} x \, dx$



$A_2 = \frac{1}{2}bh$
$= \frac{1}{2}(3)(3) = 4.5$

$A_1 = \frac{1}{2}bh$
$= \frac{1}{2}(2)(2)$
$= 2$

$A_T = 2 + 4.5 = 6.5$

$$-\frac{x^2}{2}\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_{0}^{3}$$

$$\left(0 - \frac{-4}{2}\right) + \left(\frac{9}{2} - 0\right) =$$

$$2 + \frac{9}{2} = 6.5$$