5.5 and 5.6 u-substitution

Ex. $\int x^3(x^4+5)^9\,dx =$

let $u = x^4+5$

$\frac{du}{dx} = 4x^3$

$dx = \frac{du}{4x^3}$

$\int \cancel{x^3}\cdot u^9 \cdot \frac{du}{4\cancel{x^3}} =$

$\frac{1}{4}\int u^9\,du = \frac{1}{4}\cdot\frac{u^{10}}{10} + C$

$= \frac{u^{10}}{40} + C = \frac{(x^4+5)^{10}}{40} + C$

Ex. $\int x^2(x^3-1)^4\,dx =$

let $u = x^3-1$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$\int \cancel{x^2}\cdot u^4 \cdot \frac{du}{3\cancel{x^2}} =$

$\frac{1}{3}\int u^4\,du = \frac{1}{3}\cdot\frac{u^5}{5} + C$

$= \frac{u^5}{15} + C = \frac{(x^3-1)^5}{15} + C$

Ex. $\int \tan^2 x \sec^2 x\,dx$

let $u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int u^2 \cdot \cancel{\sec^2 x} \cdot \frac{du}{\cancel{\sec^2 x}} =$

$\int u^2\,du = \frac{u^3}{3} + C = \frac{\tan^3 x}{3} + C$

Ex. $\displaystyle\int_{x=\pi/2}^{x=\pi} \cos(2x-\pi)\,dx$

let $u = 2x - \pi$

$\dfrac{du}{dx} = 2$

$dx = \dfrac{du}{2}$

$$\int_{u=0}^{u=\pi} \cos u \cdot \frac{du}{2} =$$

$$\frac{1}{2}\int_0^{\pi} \cos u \, du = \frac{1}{2}\Big[\sin u\Big]_0^{\pi} = \frac{1}{2}(\sin\pi - \sin 0)$$

$$= \frac{1}{2}(0-0) = \frac{1}{2}(0)$$

$$= 0$$

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NORMAL FLOAT AUTO REAL RADIAN CL
fnInt(cos(2X-π),X,π/2,π)
                    -8.8302817E-14
```

≈ 0

$$\int_3^{\sqrt{14}} \frac{2x}{\sqrt{x^2-5}}\,dx =$$

let $u = x^2 - 5$

$\dfrac{du}{dx} = 2x$

$dx = \dfrac{du}{2x}$

$$\int_4^9 \frac{\cancel{2x}}{\sqrt{u}} \cdot \frac{du}{\cancel{2x}} =$$

$$\int_4^9 u^{-\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}}\Bigg|_4^9 = 2\sqrt{u}\,\Big|_4^9 = 2(3-2)$$

$$= 2$$

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NORMAL FLOAT AUTO REAL RADIAN CL
fnInt(2X/√(X²-5),X,3,√(14)
)
                                 2
```

$$\int x(x-1)^4\,dx$$

let $u = x-1 \Rightarrow x = u+1$

$\frac{du}{dx} = 1$

$dx = du$

$$\int x \cdot u^4 \cdot du =$$

$$\int u^4(u+1)\,du =$$

$$\int (u^5+u^4)\,du = \frac{u^6}{6} + \frac{u^5}{5} + C = \frac{(x-1)^6}{6} + \frac{(x-1)^5}{5} + C$$

$$\int \frac{x}{\sqrt{x-1}}\,dx$$

let $u = x-1 \Rightarrow x = u+1$

$\frac{du}{dx} = 1$

$dx = du$

$$\int \frac{x}{\sqrt{u}}\,du =$$

$$\int \frac{u+1}{u^{1/2}}\,du = \int \left(\frac{u}{u^{1/2}} + \frac{1}{u^{1/2}}\right) du$$

$$= \int \left(u^{1/2} + u^{-1/2}\right) du = \frac{u^{3/2}}{3/2} + \frac{u^{1/2}}{1/2} + C$$

$$= \frac{2}{3}u^{3/2} + 2u^{1/2} + C$$

$$= \frac{2}{3}(x-1)^{3/2} + 2(x-1)^{1/2} + C$$

Jun 29-10:03 AM

$$\int \cos(kx)\,dx \qquad \text{let } u = kx$$

$$\frac{du}{dx} = k$$

$$\int \cos u \cdot \frac{du}{k} = \qquad dx = \frac{du}{k}$$

$$\frac{1}{k}\int \cos u\,du = \frac{\sin u}{k} + C = \frac{\sin kx}{k} + C$$

Similarly,

$$\int \sin kx\,dx = -\frac{\sin kx}{k} + C$$

Jul 14-6:55 PM

$$\int \sqrt{x}\,\sin^2\left(x^{3/2}-1\right)dx$$

let $u = x^{3/2} - 1$

$\frac{du}{dx} = \frac{3}{2}x^{1/2}$

$dx = \frac{2}{3\sqrt{x}}\,du$

$\cos 2x = 1 - 2\sin^2 x$

$\cos 2x - 1 = -2\sin^2 x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$$\int \cancel{\sqrt{x}}\,\sin^2 u \cdot \frac{2}{3\cancel{\sqrt{x}}}\,du =$$

$$\frac{2}{3}\int \sin^2 u\,du =$$

$$\frac{2}{3}\int \frac{1-\cos 2u}{2}\,du =$$

$$\frac{1}{3}\int (1-\cos 2u)\,du = \frac{1}{3}\left[u - \frac{\sin 2u}{2}\right] + C$$

$$= \frac{1}{3}\left[\left(x^{3/2}-1\right) - \frac{\sin\left(2\left(x^{3/2}-1\right)\right)}{2}\right] + C$$

$$= \frac{x^{3/2}-1}{3} - \frac{\sin\left(2x^{3/2}-2\right)}{6} + C$$

$$\int_0^{\sqrt{2\pi}/2} x\cos^2\left(x^2\right)dx$$

$\left(\frac{\sqrt{2\pi}}{2}\right)^2 = \frac{2\pi}{4} = \frac{\pi}{2}$

let $u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\cos 2\theta = 2\cos^2\theta$

$\cos^2\theta = \frac{\cos 2\theta}{2}$

$$\int_0^{\pi/2} \cancel{x}\cos^2 u \cdot \frac{du}{2\cancel{x}} = \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos^2 u\,du =$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\cos 2u + 1}{2}\,du = \frac{1}{4}\int_0^{\frac{\pi}{2}} (\cos 2u + 1)\,du$$

$$= \frac{1}{4}\left[\frac{\sin 2u}{2} + u\right]_0^{\pi/2}$$

$$= \frac{1}{4}\left[\left(0 + \frac{\pi}{2}\right) - (0+0)\right]$$

```
NORMAL FLOAT AUTO REAL RADIAN CL
π/8
                          .3926990817
fnInt(Xcos(X²)²,X,0,√(2π)/
2)
                          .3926990817
```

Jun 29-10:03 AM

$$\int_{x=\frac{\pi}{2}}^{x=\pi} \sin x \cos x \, dx =$$

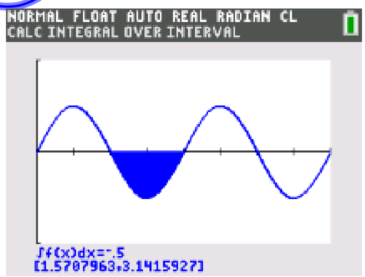
let $u = \sin x$

$$\frac{du}{dx} = \cos x$$

$$dx = \frac{du}{\cos x}$$

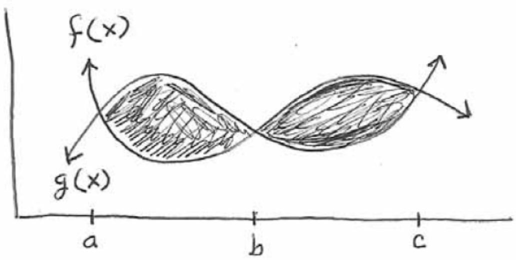
$$\int_{u=1}^{u=0} u \cdot \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}} = \int_1^0 u \, du$$

$$= \frac{u^2}{2}\Big|_1^0 = 0 - \frac{1}{2} = -\frac{1}{2}$$



5.6

Area bounded by curves



$$\text{Area of shaded region} = \int_a^b [g(x) - f(x)]\,dx + \int_b^c [f(x) - g(x)]\,dx$$

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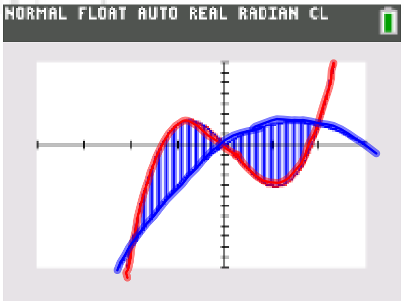
Ex. Find the area bdd by $\begin{cases} y_1 = -x^2+3x \\ y_2 = 2x^3-x^2-5x \end{cases}$



$2x^3-x^2-5x = -x^2+3x$

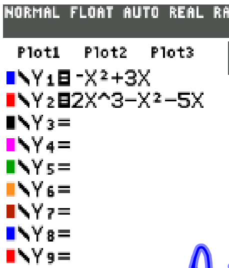
$2x^3-8x=0$

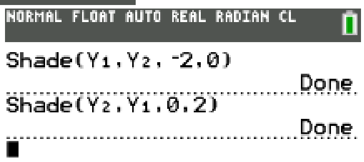
$2x(x^2-4)=0$

$2x(x+2)(x-2)=0$

intersections:

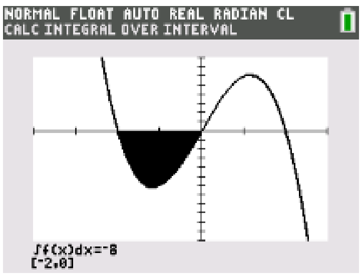
$x=0, -2, 2$

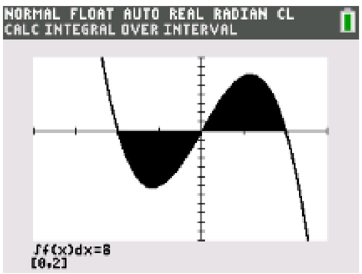


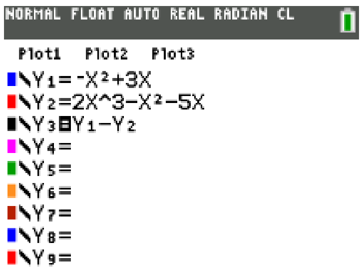


$$A = \int_{-2}^{0}\left[2x^3-x^2-5x-(-x^2+3x)\right]dx \; +$$

$$\int_{0}^{2}\left[-x^2+3x-(2x^3-x^2-5x)\right]dx = 16$$

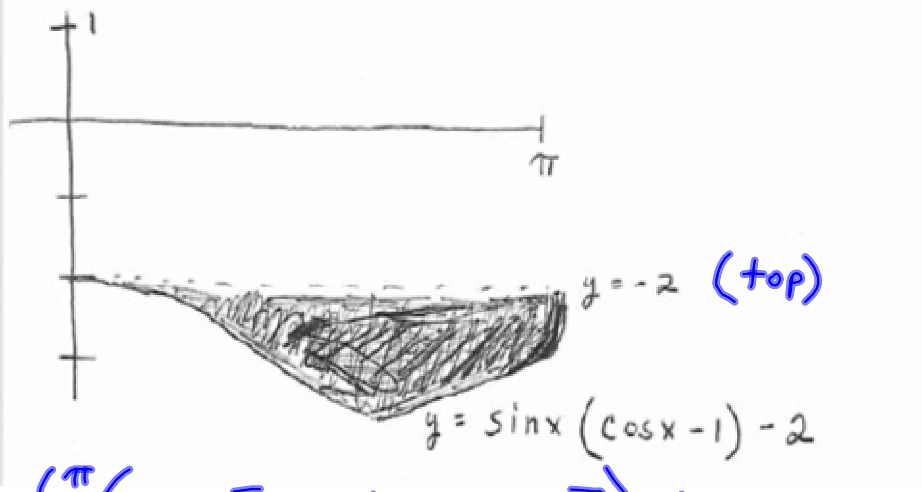






$|-8| + 8 = 16$

Find the area of the shaded region



$$\int_0^{\pi}\left(-2-\left[\sin x\,(\cos x-1)-2\right]\right)dx$$

$$\int_0^{\pi}-\sin x\,(\cos x-1)\,dx = \int_0^{\pi}\sin x\,(1-\cos x)\,dx$$

let $u = 1-\cos x$

$\frac{du}{dx} = \sin x$

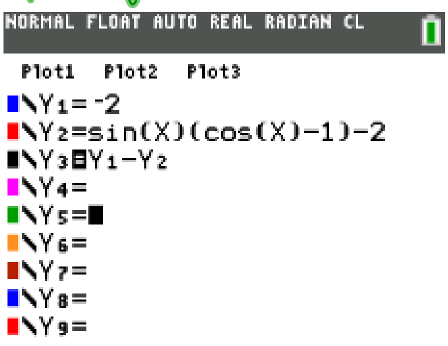
$dx = \frac{du}{\sin x}$

$$= \int_0^{2} \cancel{\sin x}\cdot u \cdot \frac{du}{\cancel{\sin x}}$$

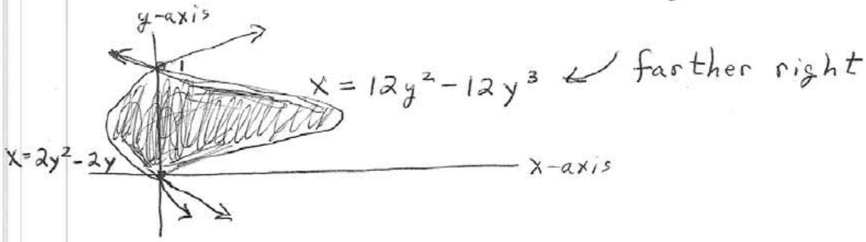
$$= \int_0^{2} u\,du = \frac{u^2}{2}\Big|_0^2 = 2-0$$

$$= 2$$

turn equal signs off Y_1 & Y_2



Find the area of the shaded region



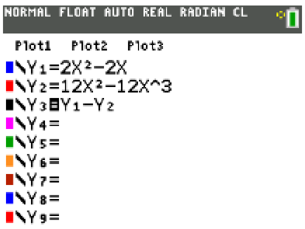
$$A = \int_0^1 \left[(12y^2 - 12y^3) - (2y^2 - 2y)\right] dy$$

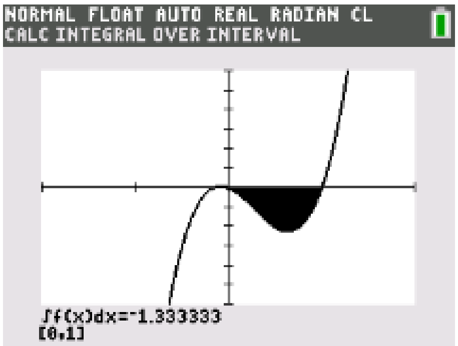
$$= \int_0^1 (10y^2 - 12y^3 + 2y)\, dy$$

$$= \frac{10y^3}{3} - \frac{12y^4}{4} + \frac{2y^2}{2} \Big|_0^1$$

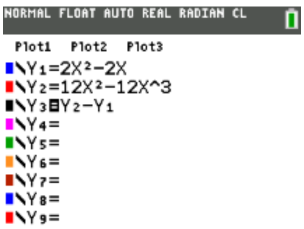
$$= \frac{10}{3}y^3 - 3y^4 + y^2 \Big|_0^1 = \left(\frac{10}{3} - 3 + 1\right) - 0$$

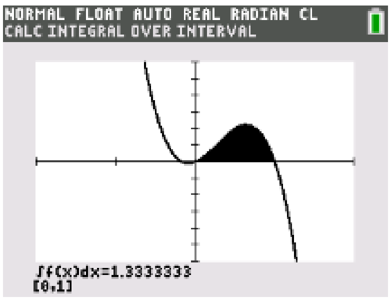
$$= \frac{10}{3} - \frac{9}{3} + \frac{3}{3} = \frac{4}{3} = 1.\overline{3}$$





$$|-1.\overline{3}| = 1.\overline{3} = \frac{4}{3}$$





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