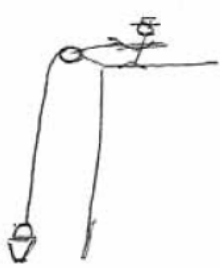
6.5 Work

work = force x distance $W = Fd$

Common units
English: feet*pounds = ft-lb
Metric: Newtons*meters = Nm=joule

Ex. A 50 pound bucket is lifted from the ground to the top of a building by pulling 30 feet of rope. Find the amount of work done to get the bucket to the top if the weight of the rope is



a) negligible (force does not vary)

i) without calculus

$W = Fd = (50\ lb)(30\ ft) = 1500\ ft\text{-}lb$

ii) with calculus

$$W = \int_0^{30} 50\,dy = 50y\Big|_0^{30} = 50(30) - 0 = 1500\ ft\text{-}lb$$

b) 0.25 pounds per foot (force varies)

let y = amount of rope pulled in

$$F = 50 + 0.25(30 - y)$$

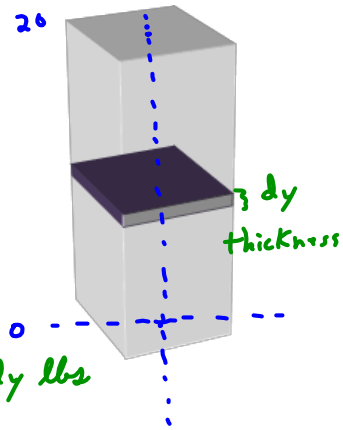
$$= 50 + 7.5 - 0.25y = 57.5 - 0.25y$$

$$W = \int_0^{30} (57.5 - 0.25y)\,dy = 57.5y - \frac{0.25y^2}{2}\Big|_0^{30}$$

$$= 57.5(30) - \frac{0.25(30)^2}{2} = 1612.5\ ft\text{-}lb$$

Ex. A rectangular tank with a 10 foot by 12 foot base with 20 foot depth is used to catch runoff water (water has a density of 62.4 pounds per cubic foot). If its top is at ground level

ρ density



a) how much work does it take to empty a full tank by pumping the water back to the ground?

$F = V\rho$

$A_{rect.} = (10\,ft)(12\,ft) = 120\,ft^2$

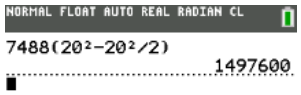
$V_{slab} = (120\,ft^3)\,dy$

$F = (62.4\,lbs/ft^3)(120\,ft^3)\,dy = 7488\,dy\ lbs$

$d = (20-y)\,ft$: height to attain minus initial height of slab/rectangle

$$W = \int_0^{20} (7488\,dy)(20-y) = 7488\int_0^{20} (20-y)\,dy$$

$$= 7488\left[20y - \frac{y^2}{2}\right]_0^{20} = 1,497,600\ ft\text{-}lb$$



b) if it is pumped out with a $^5/_{11}$ horsepower motor, how long will it take to empty a full tank? 1 hp is 550 ft-lb/sec. $\left(\frac{5}{11}\cdot 550 = 250\right)$

$$P = \frac{W}{t} \Rightarrow t = \frac{W}{P} = \frac{1,497,600\ ft\text{-}lb}{250\ ft\text{-}lb/sec}$$

$= 5990$ sec

$\approx$ 1 hr 40 min $\approx$ 100 minutes

c) How long will it take to empty half the tank?

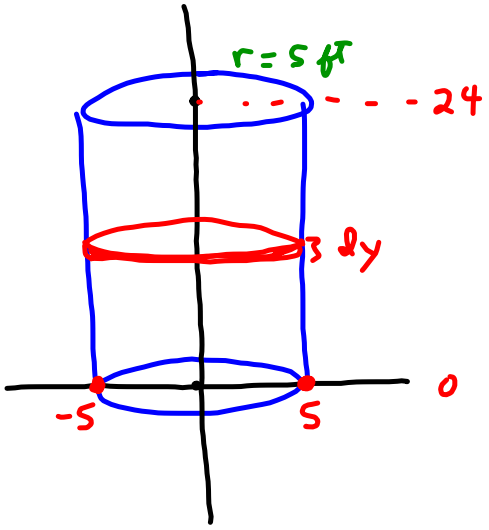
$$= 7488\left[20y - \frac{y^2}{2}\right]_{10}^{20} = 1497600 - 7488(200-50)$$

$= 374,400\ ft\text{-}lb$

$$t = \frac{W}{P} = \frac{374,400}{250} \approx 1500\ sec$$

$\approx$ 25 minutes

A vertical right circular cylindrical tank measures 24 ft high and 10 ft in diameter. It is full of oil weighing 60 lb/ft^3. How much work does it take to pump the oil to the level of the top of the tank? Give your answer to the nearest ft • lb.



$A = \pi r^2$
$= 25\pi$

$V_{disc} = 25\pi\, dy$

$F = V\rho$
$= 25\pi\, dy\,(60)$
$= 1500\pi\, dy$

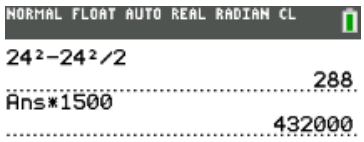
distance = $24 - y$

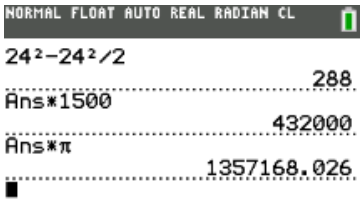
$$W = \int_0^{24} (1500\pi\, dy)(24-y) = 1500\pi \int_0^{24} (24-y)\, dy$$

$$= 1500\pi \left[24y - \frac{y^2}{2}\right]_0^{24}$$

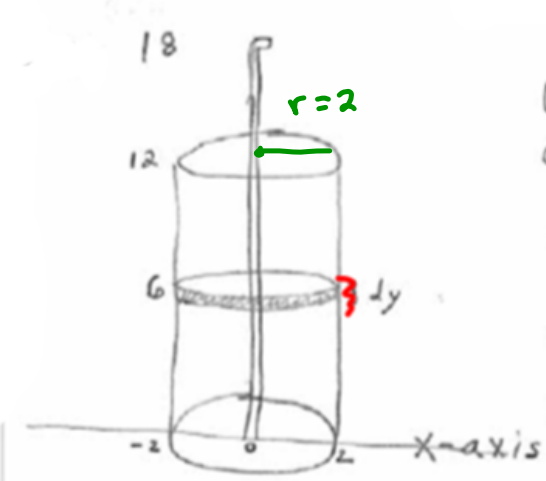
$$= 432{,}000\pi \text{ ft-lb}$$

$$\approx 1{,}357{,}168 \text{ ft-lb}$$





A vertical storage tank is a right circular cylinder 12 feet high and 4 feet in diameter. If the tank is half full of crude oil which weighs 53.8 lb per cubic foot, find the work done in emptying it through



We are pumping out circles (or discs).

Area of circle $= \pi r^2 = \pi(2)^2 = 4\pi$

Volume of disc $= 4\pi\, dy$

Force = weight $= 4\pi\, dy\,(53.8) = 215.2\pi\, dy$

$d = 18 - y$ where $y = 0$ to 6

$$W = \int_0^6 215.2\pi\, dy\,(18-y) = 215.2\pi \int_0^6 (18-y)\, dy$$

$$215.2\pi \left[18y - \frac{y^2}{2}\right]_0^6 \approx$$

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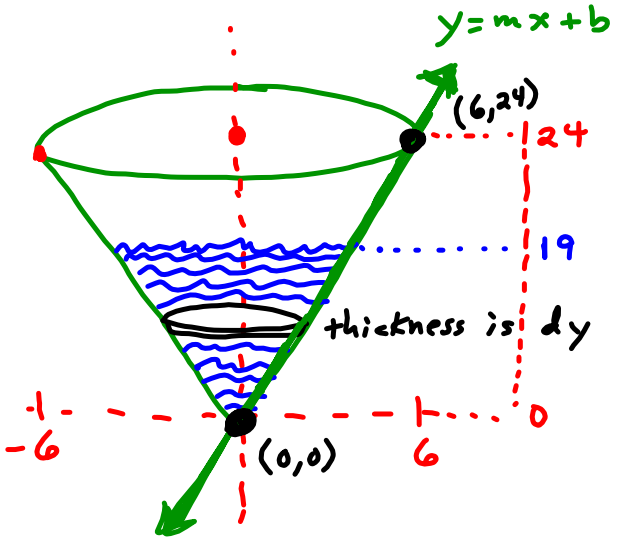
18(6)−6²/2 → 90

Ans*215.2 → 19368

Ans*π → 60846.36651

$19{,}368\pi$ ft-lb or 60846 ft-lb

A conical container of radius 6 ft and height 24 ft is filled to a height of 19 ft of a liquid weighing 50.6 lb / ft^3. How much work will it take to pump the contents to the rim?



$m = \frac{\Delta y}{\Delta x} = \frac{24-0}{6-0} = 4$

$y = 4x$ (radius = x)

$x = \frac{y}{4}$

$A_{circle} = \pi r^2$

$= \pi \left(\frac{y}{4}\right)^2$

$= \frac{\pi y^2}{16}$

$d = 24 - y$, $y = 0$ to 19

$V_{disc} = \frac{\pi y^2}{16} dy$

$F = Vp = \frac{\pi y^2}{16} dy\,(50.6)$

$$W = \int_0^{19} \frac{\pi y^2}{16} dy\,(50.6)\,(24-y) \approx 221{,}475 \text{ ft-lb}$$

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fnInt(50.6πX²/16(24-X),X,0,19)

221474.9288