Find $\int_{x=\frac{1}{2}}^{x=\frac{3}{2}} x \cdot 5^{x^2}\, dx =$

$$\int a^x = \frac{a^x}{\ln a} + C$$

let $u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$$\frac{1}{2}\int_{\frac{1}{4}}^{9/4} 5^u\, du = \frac{1}{2}\left[\frac{5^u}{\ln 5}\right]_{\frac{1}{4}}^{9/4} =$$

$$\frac{1}{2\ln 5}\left[5^u\right]_{\frac{1}{4}}^{9/4} = \frac{1}{\ln 25}\left[5^{9/4} - 5^{\frac{1}{4}}\right] =$$

$$\frac{1}{\ln 25} \cdot 5^{\frac{1}{4}}\left(5^{8/4} - 1\right) = \frac{\sqrt[4]{5}}{\ln 25}(24) = \frac{24\sqrt[4]{5}}{\ln 25}$$

Find $\frac{dy}{dx}$ if

$$\frac{d}{dx}\log_a u = \frac{u'}{u \ln a}$$

$$y = \log_{\sqrt{2}} \underbrace{\sec(2x+1)}_{u}$$

$$\frac{dy}{dx} = \frac{\sec(2x+1)\tan(2x+1)\cdot 2}{\sec(2x+1)\cdot \ln\sqrt{2}} = \frac{2\tan(2x+1)}{\ln 2^{1/2}}$$

$$= \frac{2\tan(2x+1)}{\frac{1}{2}\ln 2}$$

$$= \frac{4\tan(2x+1)}{\ln 2}$$

Find $\frac{dy}{dx}$ if $y = x^x$

$\ln y = \ln x^x$

Rewrite

$\ln y = x \ln x$

$\frac{y'}{y} = x \cdot \frac{1}{x} + \ln x \cdot 1$

$\frac{y'}{y} = 1 + \ln x$; $y' = y(1+\ln x) = x^x(1+\ln x)$

Find $\frac{dy}{dx}$ if $y = (\sin x)^x$

$\ln y = \ln(\sin x)^x$

$\ln y = x \ln(\sin x)$

$\frac{y'}{y} = x \cdot \frac{\cos x}{\sin x} + \ln(\sin x) \cdot 1$

$\frac{y'}{y} = x \cot x + \ln \sin x$

$y' = (\sin x)^x (x \cot x + \ln \sin x)$