7.6 Inverse Trig Functions

① $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \qquad \left(-1 < \frac{x}{a} < 1\right)$

② $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

③ $\displaystyle\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

$\displaystyle = \frac{1}{a}\cos^{-1}\left|\frac{a}{x}\right| + C \qquad \left(-1 < \frac{a}{x} < 1\right)$

Ex. $\displaystyle\int \frac{7}{4+4x^2}\,dx = \int \frac{7\,dx}{4(1+x^2)} =$

$a=1$

$\displaystyle\frac{7}{4}\int\frac{dx}{1+x^2} \overset{②}{=} \frac{7}{4}\cdot\frac{1}{1}\cdot\tan^{-1}\left(\frac{x}{1}\right) + C$

$\displaystyle = \frac{7}{4}\tan^{-1}x + C$

Ex. $\displaystyle\int_{\frac{2}{\sqrt{3}}}^{\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}} = \cos^{-1}\frac{1}{x}\Bigg|_{2/\sqrt{3}}^{\sqrt{2}} =$

$\frac{\sqrt{2}}{2}$

$\displaystyle\cos^{-1}\frac{1}{\sqrt{2}} - \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{4} - \frac{\pi}{6}$

$\displaystyle = \frac{\pi}{12}$

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NORMAL FLOAT AUTO a+bi RADIAN CL
fnInt(1/(X√(X²-1)),X,2/√(3
),√(2))
                .2617993878
π/12
                .2617993878
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Review of completing the square (CTS)

Fill in the blanks so that the polynomial is a perfect square

① $x^2 - 8x + \underline{16} = (x - \underline{4})^2$

② $x^2 + 16x + \underline{64} = (x + \underline{8})^2$

③ $x^2 - 7x + \underline{\tfrac{49}{4}} = (x - \underline{\tfrac{7}{2}})^2$

Solve $\frac{3x^2}{3} - \frac{6x}{3} - \frac{7}{3} = \frac{0}{3}$ using CTS

NORMAL FLOAT AUTO REAL RADIAN CL
X=
3+/-√30

3
PRESS ENTER
WHEN FINISHED

$x^2 - 2x + \underline{1} = \frac{7}{3} + \underline{\tfrac{3}{3}}$

$(x-1)^2 = \pm\frac{10}{3}$

$x - 1 = \frac{\pm\sqrt{10}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\pm\sqrt{30}}{3} \Rightarrow x = 1 \pm \frac{\sqrt{30}}{3}$

$= \frac{3}{3} \pm \frac{\sqrt{30}}{3} = \frac{3 \pm \sqrt{30}}{3}$

① $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \quad \left(-1 < \frac{x}{a} < 1\right)$

② $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

③ $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + C$

$= \frac{1}{a} \cos^{-1}\left|\frac{a}{x}\right| + C \quad \left(-1 < \frac{a}{x} < 1\right)$

$$\int \frac{dx}{x^2 + 6x + 18} = \int \frac{dx}{[x^2 + 6x + 9] - 9 + 18}$$

let $u = x+3$

$\frac{du}{dx} = 1$

$du = dx$

$$= \int \frac{dx}{(x+3)^2 + 9} = \int \frac{dx}{3^2 + (x+3)^2}$$

$$= \int \frac{du}{3^2 + u^2} = \frac{1}{3} \tan^{-1} \frac{u}{3} + C$$

$$= \frac{1}{3} \tan^{-1} \frac{(x+3)}{3} + C$$

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① $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \quad \left(-1 < \frac{x}{a} < 1\right)$

② $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

③ $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

$= \frac{1}{a}\cos^{-1}\left|\frac{a}{x}\right| + C \quad \left(-1 < \frac{a}{x} < 1\right)$

$$\int \frac{dx}{\sqrt{4x-x^2}} =$$

$4x - x^2 = -x^2 + 4x$

$= -(x^2 - 4x + \underline{4}) + 4$

$= -(x-2)^2 + 4$

$= 4 - (x-2)^2$

$= 2^2 - (x-2)^2$

$$\int \frac{dx}{\sqrt{2^2 - (x-2)^2}} =$$

let $u = x-2$

$\frac{du}{dx} = 1$

$du = dx$

$$\int \frac{du}{\sqrt{2^2-u^2}} = \sin^{-1}\left(\frac{u}{2}\right) + C$$

$$= \sin^{-1}\frac{(x-2)}{2} + C$$

① $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \qquad \left(-1 < \frac{x}{a} < 1\right)$

② $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

③ $\displaystyle\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C = \frac{1}{a}\cos^{-1}\left|\frac{a}{x}\right| + C \qquad \left(-1 < \frac{a}{x} < 1\right)$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{16\,dx}{4x^2+4x+5} =$$

$$4x^2+4x+5 =$$
$$4\left(x^2+x+\frac{1}{4}\right)+5-1$$
$$4\left(x+\frac{1}{2}\right)^2+4 =$$
$$4\left[\left(x+\frac{1}{2}\right)^2+1\right]$$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{16\,dx}{4\left[\left(x+\frac{1}{2}\right)^2+1\right]} =$$

$$4\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{1^2+\left(x+\frac{1}{2}\right)^2} =$$

let $u = x+\frac{1}{2}$

$\frac{du}{dx} = 1$

$dx = du$

$$4\int_{0}^{1} \frac{du}{1^2+u^2} = 4\tan^{-1}u\Big|_0^1 = 4\left(\tan^{-1}1 - \tan^{-1}0\right)$$
$$= 4\left(\frac{\pi}{4} - 0\right) = \pi$$

NORMAL FLOAT AUTO a+bi DEGREE MP

$\int_{-1/2}^{1/2} (16/(4X^2+4X+5))dX$

3.141592654

NORMAL FLOAT AUTO a+bi DEGREE CL

fnInt(16/(4X²+4X+5),X,-1/2,1/2)

3.141592654

① $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \quad \left(-1 < \frac{x}{a} < 1\right)$

② $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

③ $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$
$= \frac{1}{a}\cos^{-1}\left|\frac{a}{x}\right| + C \quad \left(-1 < \frac{a}{x} < 1\right)$

$$\int_{\ln\sqrt{3}}^{\ln 3} \frac{36e^x}{9+e^{2x}}\,dx =$$

let $u = e^x$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$= \frac{du}{u}$

$$\int_{e^{\ln\sqrt{3}}}^{e^{\ln 3}} \frac{36\cancel{u}}{9+u^2}\cdot\frac{du}{\cancel{u}} =$$

$$\int_{\sqrt{3}}^{3} \frac{36\,du}{3^2+u^2} = 36\left(\frac{1}{3}\right)\tan^{-1}\left(\frac{u}{3}\right)\Bigg|_{\sqrt{3}}^{3}$$

$$= 12\left[\tan^{-1}1 - \tan^{-1}\frac{\sqrt{3}}{3}\right]$$

$$= 12\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = 3\pi - 2\pi$$

$$= \pi$$

① $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \quad \left(-1 < \frac{x}{a} < 1\right)$

② $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

③ $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$
$= \frac{1}{a}\cos^{-1}\left|\frac{a}{x}\right| + C \quad \left(-1 < \frac{a}{x} < 1\right)$

$$\int \frac{6\,dx}{\sqrt{e^{4x}-9}} = \int \frac{6}{\sqrt{u^2-3^2}} \cdot \frac{du}{2u}$$

let $u = e^{2x}$

$\frac{du}{dx} = 2e^{2x}$
$= 2u$

$dx = \frac{du}{2u}$

$$3\int \frac{du}{u\sqrt{u^2-3^2}} = 3 \cdot \frac{1}{3}\cos^{-1}\left|\frac{3}{x}\right| + C$$

$$= \cos^{-1}\left|\frac{3}{x}\right| + C$$

$=$

NORMAL FLOAT AUTO a+bi RADIAN CL
cos⁻¹(1/3)−cos⁻¹(3/4)
.5082251695

NORMAL FLOAT AUTO a+bi RADIAN CL
fnInt(6/√(e^(4X)−9),X,ln(2),ln(3))
.5082251695

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① $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \quad \left(-1 < \frac{x}{a} < 1\right)$

② $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

③ $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

$= \frac{1}{a}\cos^{-1}\left|\frac{a}{x}\right| + C \quad \left(-1 < \frac{a}{x} < 1\right)$

$$\int \frac{3\,dx}{x\sqrt{16x^6-49}} =$$

let $u = \sqrt{16x^6} = 4x^3$; $u^2 = 16x^6$

let $u = 4x^3$

$\frac{du}{dx} = 12x^2$

$dx = \frac{du}{12x^2}$

$$\int \frac{3\,du}{12x^2 \cdot x\sqrt{u^2-7^2}} = \int \frac{3\,du}{12x^3\sqrt{u^2-7^2}}$$

$$\int \frac{du}{4x^3\sqrt{u^2-7^2}} = \int \frac{du}{u\sqrt{u^2-7^2}} = \frac{1}{7}\cos^{-1}\left|\frac{7}{u}\right| + C$$

$$= \frac{1}{7}\cos^{-1}\left|\frac{7}{4x^3}\right| + C$$

Jul 24-11:04 AM

7.7 Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}, \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}, \qquad \coth x = \frac{\cosh x}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}, \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

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NORMAL FLOAT AUTO a+bi RADIAN CL
sinh(2)
                    3.626860408
(e^(2)-e^(-2))/2
                    3.626860408
```

$$\frac{d}{dx}(\sinh x) = \cosh x \ , \quad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x \ , \quad \frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x \ , \quad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\int \sinh x \, dx = \cosh x + C \qquad \int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$$

$$\int \cosh x \, dx = \sinh x + C \qquad \int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$$

$$\int \operatorname{sech}^2 x \, dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x \, dx = -\coth x + C$$

$\frac{d}{dx}(\sinh x) = \cosh x$, $\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$, $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$, $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

Ex. $\frac{d}{dx}\left[\sinh(5x^3+x-1)\right] =$

$(15x^2+1)\cosh(5x^3+x-1)$

Ex. $\frac{d}{dx}\left[\operatorname{sech}(2-x^2)\right] =$

$-2x\left(-\operatorname{sech}(2-x^2)\tanh(2-x^2)\right) =$

$2x\operatorname{sech}(2-x^2)\tanh(2-x^2)$

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$\int \sinh x\,dx = \cosh x + C$

$\int \cosh x\,dx = \sinh x + C$

$\int \operatorname{sech}^2 x\,dx = \tanh x + C$

$\int \operatorname{csch}^2 x\,dx = -\coth x + C$

$\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + C$

$\int \operatorname{csch} x \coth x\,dx = -\operatorname{csch} x + C$

Ex. $\displaystyle\int_0^{\ln 5} \cosh x\,dx = \sinh x \Big|_0^{\ln 5} =$

$$\frac{e^x - e^{-x}}{2}\Bigg|_0^{\ln 5} = \frac{e^{\ln 5} - e^{-\ln 5}}{2} - \frac{e^0 - e^0}{2}$$

$\ln \frac{1}{5}$

$$= \frac{5\left(5 - \frac{1}{5}\right)}{5(2)} = \frac{25-1}{10}$$

$$= \frac{24}{10} = 2.4$$

$\int \sinh x\,dx = \cosh x + C$

$\int \cosh x\,dx = \sinh x + C$

$\int \operatorname{sech}^2 x\,dx = \tanh x + C$

$\int \operatorname{csch}^2 x\,dx = -\coth x + C$

$\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + C$

$\int \operatorname{csch} x \coth x\,dx = -\operatorname{csch} x + C$

$$\int \tanh(4x)\,dx = \int \frac{\sinh(4x)}{\cosh(4x)}\,dx$$

let $u = \cosh(4x)$

$\frac{du}{dx} = 4\sinh(4x)$

$dx = \frac{du}{4\sinh(4x)}$

$= \frac{1}{4}\int \frac{du}{u} = \frac{1}{4}\ln|u| + C$

$= \frac{1}{4}\ln|\cosh(4x)| + C$

$$\int 2e^x \sinh x\,dx = 2\int e^x \cdot \frac{e^x - e^{-x}}{2}\,dx =$$

$\int e^x(e^x - e^{-x})\,dx = \int (e^{2x} - e^0)\,dx$

$= \int (e^{2x} - 1)\,dx$

$= \frac{e^{2x}}{2} - x + C$

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