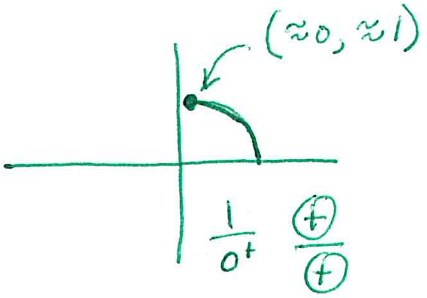
Review Part II

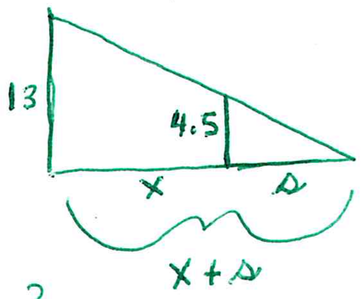
① Find the limit if it exists.

a) $$\lim_{x\to 4}\left[\frac{\sqrt{5-x}-1}{4-x}\cdot\frac{\sqrt{5-x}+1}{\sqrt{5-x}+1}=\frac{(5-x)-1}{(4-x)(\sqrt{5-x}+1)}=\frac{\cancel{4-x}\cdot 1}{\cancel{(4-x)}(\sqrt{5-x}+1)}=\frac{1}{\sqrt{5-x}+1}\right]=\frac{1}{1+1}=\boxed{\frac{1}{2}}$$

b) $$\lim_{x\to\frac{\pi}{2}^-}\left(\sec x=\frac{1}{\cos x}\right)=\boxed{\infty}$$



② A 4.5 ft girl runs 4 ft/sec from a lamppost that is 13 ft tall. At what rate is the shadow changing when she is 17 ft from the lamppost?



$\frac{ds}{dt}=?$

$\frac{dx}{dt}=4$ ft/sec

$$\frac{13}{x+s}=\frac{4.5}{s}\Rightarrow$$

$$13s=4.5x+4.5s\Rightarrow$$

$$8.5s=4.5x\Rightarrow$$

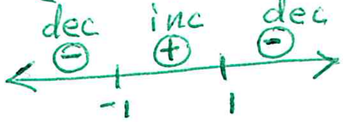
$$8.5\frac{ds}{dt}=4.5\frac{dx}{dt}\Rightarrow$$

$$\frac{ds}{dt}=\frac{4.5(4)}{8.5}=\boxed{\frac{36}{17}\text{ ft/sec}}$$

③ Find the local extrema and inflection point for $f(x) = \frac{2x}{x^2+1} \Rightarrow$

$$f'(x) = \frac{(x^2+1)(2) - 2x(2x)}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2} = \frac{2(1-x^2)}{(x^2+1)^2}$$

CV: $-1, 1$

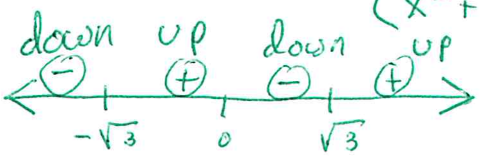


rel min @ $(-1,-1)$
rel max @ $(1, 1)$

$$f''(x) = \frac{(x^2+1)^2(-4x) - 2(x^2+1)(2x)(2-2x^2)}{(x^2+1)^4}$$

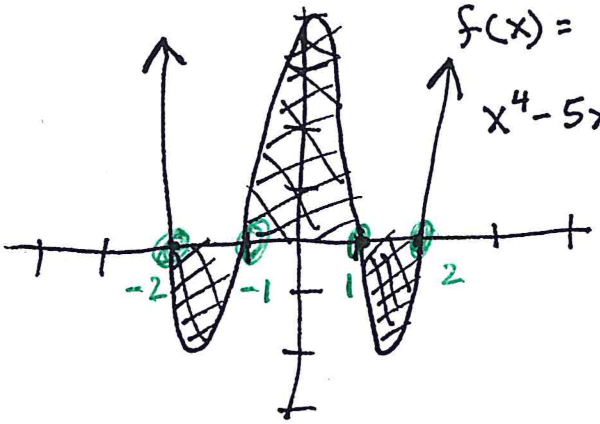
$$= \frac{-4x(x^2+1)(3-x^2)}{(x^2+1)^4}$$

CV'': $0, -\sqrt{3}, \sqrt{3}$



infl. pt $\left(-\sqrt{3}, \frac{-\sqrt{3}}{2}\right), (0,0), \left(\sqrt{3}, \frac{\sqrt{3}}{2}\right)$

④ Find the area



$f(x) = x^4 - 5x^2 + 4$

$$A = \int_{-1}^{1} f(x)\,dx + 2\int_{2}^{1} f(x)\,dx$$

$$= \frac{76}{15} + 2\left(\frac{22}{15}\right)$$

$$= \boxed{8}$$

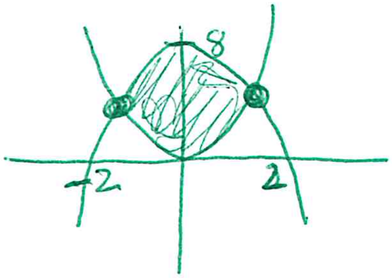
⑤ Find $\int_{-\frac{\pi}{2}}^{\pi} 2\sin^7 x \cos x \, dx =$

let $u = \sin x$

$\frac{du}{dx} = \cos x$

$$2\int_{-1}^{0} u^7 \, du = 2\cdot\frac{u^8}{8}\Big|_{-1}^{0} = \frac{u^8}{4}\Big|_{-1}^{0} = 0 - \frac{1}{4} = \boxed{-\frac{1}{4}}$$

⑥ Find area enclosed by $y = 8 - x^2$, $y = x^2$



$$A = 2\int_{0}^{2} (8 - 2x^2)\,dx = 2\left(8x - \frac{2x^3}{3}\right)\Big|_{0}^{2}$$

$$= 2\left(16 - \frac{16}{3}\right) = 2\left(\frac{32}{3}\right) = \boxed{\frac{64}{3}}$$

⑦ Find $\int \frac{x^2}{x^3+1}\,dx = \frac{1}{3}\int \frac{du}{u} =$

let $u = x^3 + 1$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$$\frac{1}{3}\ln|u| + c = \boxed{\frac{1}{3}\ln|x^3+1| + c}$$

⑧ Find $\frac{dy}{dx}$ if $y = \log_{14} \frac{x^3\sqrt[4]{x-2}}{(x+1)^7\sqrt{x-1}}$

$$y = 3\log_{14} x + \frac{1}{4}\log_{14}(x-2) - 7\log_{14}(x+1) - \frac{1}{2}\log_{14}(x-1)$$

$$\boxed{y' = \frac{1}{\ln 14}\left(\frac{3}{x} + \frac{1}{4(x-2)} - \frac{7}{x+1} - \frac{1}{2(x-1)}\right)}$$

⑨ Find y' if

$$y = \frac{x^2 \sqrt[4]{x^3+x}}{(3x+1)^{3/4}} \; ; \; \ln y = 2\ln x + \frac{1}{4}\ln(x^3+x) - \frac{3}{4}\ln(3x+1)$$

$$\frac{y'}{y} = \frac{2}{x} + \frac{3x^2+1}{4(x^3+x)} - \frac{9}{4(3x+1)}$$

$$\boxed{y' = \frac{x^2 \sqrt[4]{x^3+x}}{(3x+1)^{3/4}} \left(\frac{2}{x} + \frac{3x^2+1}{4x^3+4x} - \frac{9}{12x+4} \right)}$$

⑩ Find y' if

$$y = 7^{\sqrt[3]{x}} \Rightarrow \ln y = \sqrt[3]{x} \ln 7$$

$$\ln y = x^{\frac{1}{3}} \ln 7$$

$$\frac{y'}{y} = \frac{1}{3} x^{-2/3} \cdot \ln 7 = \frac{\ln \sqrt[3]{7}}{\sqrt[3]{x^2}}$$

$$\boxed{y' = 7^{\sqrt[3]{x}} \cdot \frac{\ln \sqrt[3]{7}}{\sqrt[3]{x^2}}}$$

⑪ $$\int \frac{dx}{\sqrt{-x^2-10x-16}} =$$

$$-x^2-10x-16 =$$

$$-(x^2+10x+\underline{25})-16+25$$

$$9-(x+5)^2$$

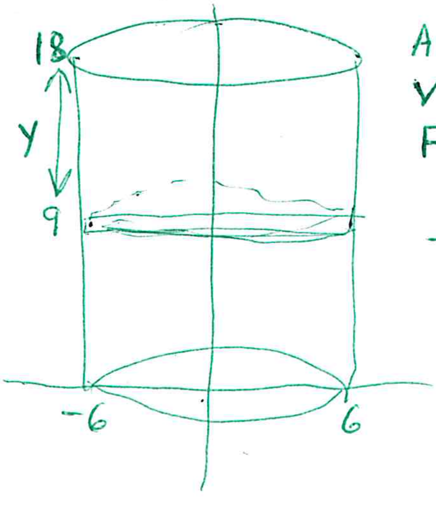
let $u = x+5$

$$\frac{du}{dx} = 1$$

$$\int \frac{du}{\sqrt{3^2-u^2}} = \sin^{-1}\frac{u}{3} + c$$

$$= \boxed{\sin^{-1}\frac{x+5}{3} + C}$$

⑫ A vertical right cylindrical tank 18 ft tall and 12 ft diameter is [illegible] full of oil weighing 58 lb/ft³. How much work does it take to empty half the tank?



$A = \pi(6)^2 = 36\pi$

$V = 36\pi\, dy$

$F = 36\pi\, dy\,(58)$

$\quad = 2088\pi\, dy$

$d = 18-y$

$$W = \int_9^{18} 2088\pi(18-y)\,dy$$

$$= \boxed{84{,}564\,\pi \text{ lb/ft}^3}$$

⑬ Find

$$\int_{\ln 2}^{\ln 7} \cosh x \, dx = \sinh x \Big|_{\ln 2}^{\ln 7} =$$

$$\frac{e^{x} - e^{-x}}{2} \Bigg|_{\ln 2}^{\ln 7} = \frac{7 - \frac{1}{7}}{2} - \frac{2 - \frac{1}{2}}{2} =$$

$$\frac{\frac{48}{7}}{2} - \frac{\frac{3}{2}}{2} = \frac{48}{14} - \frac{3}{4} =$$

$$\frac{24}{7} - \frac{3}{4} = \boxed{\frac{75}{28}}$$